Punjab Engineering College, Chandigarh
End-Term Examination

Programme: **B.E (Electrical)**
Year/Semester: 2122_2
Course Name: Electrical Machines-II
Course Code: **EL1006**
Maximum Marks: 100
Time allowed: **3 Hrs**
Course Instructor: Prof. Dhiraj Bharat
Date & Time: 13^{th} May 2022, 9:30-12:30 Hrs

- All questions are compulsory.
- Unless stated otherwise, the symbols have their usual meanings in context with subject. Assume suitably and state, additional data required, if any.
- The candidates, before starting to write the solutions, should please check the question paper for any discrepancy, and also ensure that they have been delivered the question paper of right **course code**.

<table>
<tr><th>Q.No</th><th>Question</th><th>Marks</th></tr>
<tr><td>1.A</td><td>A 220 V, three phase, two pole, 50-Hz induction motor is running at a per unit slip of 0.05. Find the following

i. The speed of the rotating magnetic field in rpm.
ii. The speed of the rotor in rpm.
iii. The slip speed of the rotor in rpm.
iv. The rotor frequency in Hz.</td><td>10</td></tr>
<tr><td>1.B</td><td>The test data on a 208-V, 60-Hz, 4-pole, Y-connected, three-phase induction motor rated at 1710 rpm are as follows:

<table>
<tr><th></th><th>No Load Test</th><th>Blocked Rotor Test</th></tr>
<tr><td>Power Input</td><td>450 W</td><td>59.4 W</td></tr>
<tr><td>Line Current</td><td>1.562 A</td><td>2.77 A</td></tr>
<tr><td>Line Voltage</td><td>208 V</td><td>27 V</td></tr>
</table>

The stator resistance (dc) between any two terminals = 2.4 Ω. Evaluate the equivalent circuit parameters of the induction motor and draw its per phase equivalent circuit with respect to the stator. Also evaluate the maximum value of the torque that the machine can supply. At what speed this maximum torque is generated?</td><td>10</td></tr>
<tr><td>2.A</td><td>A 440 V, 3-phase, 50 Hz, 6 pole, 945 rpm, delta connected induction motor has the following parameters referred to the stator:

$\mathbf{R_s = 2\ \Omega,\ R_r' = 2\ \Omega,\ X_s = 3\Omega,\ X_r' = 4\ \Omega,\ X_m = 100\ \Omega}$

When driving a load whose torque varies linearly with speed, at rated voltage it runs at rated speed. If the motor speed is controlled using stator voltage control.

i. Motor terminal voltage, current and torque at 800 rpm.
ii. Motor speed, current and torque for the terminal voltage of 280 V.</td><td>12</td></tr>
<tr><td>2.B</td><td>Write short notes on the following:

i. PMBLDC Motor.
ii. Split Phase Induction Motor.</td><td>8</td></tr>
</table>

3.A A 9-kVA, 208-V, 60-Hz, Y Connected, three phase alternator has a synchronous impedance of $0.1 + j3\ \Omega$ per phase. The field winding resistance is 25 Ω and the applied field voltage is 120-V (dc). The per phase OCC data at rated speed is as under

$i_f(A)$	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
$E_a(V)$	0	90	140	185	209	223	230	236	238	248	250

If the rotation loss is 800 W, determine the voltage regulation and efficiency of the generator when it supplies the rated load at 0.9 pf lagging. What must be the external resistance in the field winding circuit.

[10]

3.B Condition for "unity internal power factor" is a preferred condition in the operation of synchronous generators. Determine the governing dynamics for operation of unity internal power factor for a cylindrical rotor synchronous generator in terms of armature current, power factor angle, synchronous reactance and per phase terminal voltage. Also determine the absolute limit of armature current beyond which condition of unity internal power factor cannot be imposed. Neglect armature resistance.

[10]

4.A A 2200-V, 60-Hz, three-phase, Y connected, 8 pole synchronous motor has synchronous impedance of $0.8 + j8\ \Omega$. At no load, the field excitation is adjusted so that the excitation voltage is equal in magnitude to the applied voltage. When the motor is loaded the torque angle is 22^o (lagging). Compute the torque developed by the motor.

[10]

4.B A 208-V, 60-Hz, three-phase, Y-connected, salient pole synchronous motor operates at full load and draws a current of 40 A at 0.8 pf lead. The d-axis and q-axis reactance are 2.7Ω and 1.7Ω, respectively. The armature winding resistance is negligible and rotational loss is 5% of the power developed by the motor. Determine the following

i. Per phase excitation voltage $\widetilde{E_a}$.
ii. Synchronous power and salient power.
iii. Torque angle.

[10]

5.A A 208-V, three-phase, Y connected, synchronous motor takes 7.2 kW at 0.8 pf leading. The synchronous reactance of the motor is 4Ω and the armature winding resistance is negligible. Determine the new torque angle, armature current and power factor if

i. The field excitation is adjusted to decrease the excitation voltage by 50% while the load on the motor remains same.
ii. The field excitation is adjusted to obtain a unity power factor while the load on the motor remains same.

[10]

5.B A salient pole synchronous motor has negligible armature resistance. It is connected to infinite bus and giving a constant power output. Derive the differential equation indicating the governing dynamics of change of power angle with change in the induced voltage with induced voltage as independent variable and power angle as the dependent variable.

[10]

End Semester Examination Solutions

Electrical Machines - II

2021-22 - 2, DOE - 13th May 2022

Ans 1. (A) 220 V, 3∅, 2 pole, 50 Hz

$s = 0.05$

Speed of rotating MF $= \frac{120f}{P} = \frac{120 \times 50}{2} = 3000$ rpm $= N_s$

$N_m = S N_s = (1-s) N_s = 0.95 \times 3000$
$= 2850$ rpm

Slip speed = slip (PU) × Base
$= 0.05 \times 3000$
$= 150$ rpm

$f_r = sf = 0.05 \times 50 = 2.5$ Hz.

Ans 1. (B) $W_{NL} = 450$ $\quad W_{NL}/\text{phase} = \frac{450}{3} = 150$ W

$I_{NL} = 1.562$ A $= I_{NL}/\text{phase}$

$V_{NL} = 208 \Rightarrow V/\text{phase} = \frac{208}{\sqrt{3}} = 120.088$

$R_{dc} = 2.4 \Rightarrow R_s = \frac{2.4}{2} = 1.2\ \Omega$

$\frac{(V/\text{phase})^2}{R_c} = W_{NL}/\text{phase} = 150$

$\Rightarrow \frac{\left(\frac{208}{\sqrt{3}}\right)^2}{R_c} = 150 \Rightarrow R_c = \frac{(208)^2}{3 \times 150}$
$= 96.1422\ \Omega$

(1)

$Q_{NL}/\text{phase} = \sqrt{S_{NL}^2/\text{phase} - W_{NL}^2/\text{phase}}$

$= \sqrt{\left(\frac{208}{\sqrt{3}} \times 1.562\right)^2 - (150)^2} = 112.63127$ VAR

$$\frac{V_{NL/phase}^2}{X_m} = Q_{NL/phase}$$

$$\frac{\left(\frac{208}{\sqrt{3}}\right)^2}{X_m} = 112.63127$$

$$\Rightarrow X_m = 128.04\ \Omega$$

$W_{br} = 59.4\text{ W}$ $\quad W_{br/phase} = 19.8\text{ W}$

$I_{br} = 2.77\text{A} = I_{br/phase}$ $\quad V_{br/phase} = \frac{27}{\sqrt{3}} = 15.5884\text{ V}$

$V_{br} = 27\text{ V}$

$$(I_{br})^2 R_e = W_{br/phase}$$

$$(2.77)^2 R_e = 19.8 \Rightarrow R_e = 2.5805\ \Omega$$

$$\Rightarrow R_s + R_r' = 2.5805\ \Omega$$

$$\Rightarrow R_r' = 2.5805 - 1.2 = 1.3805\ \Omega$$

$$Q_{br/phase} = \sqrt{S_{br/phase}^2 - W_{br/phase}^2}$$

$$= \sqrt{\left(\frac{27}{\sqrt{3}} \times 2.77\right)^2 - (19.8)^2}$$

$$= 38.3728\text{ VAR.}$$

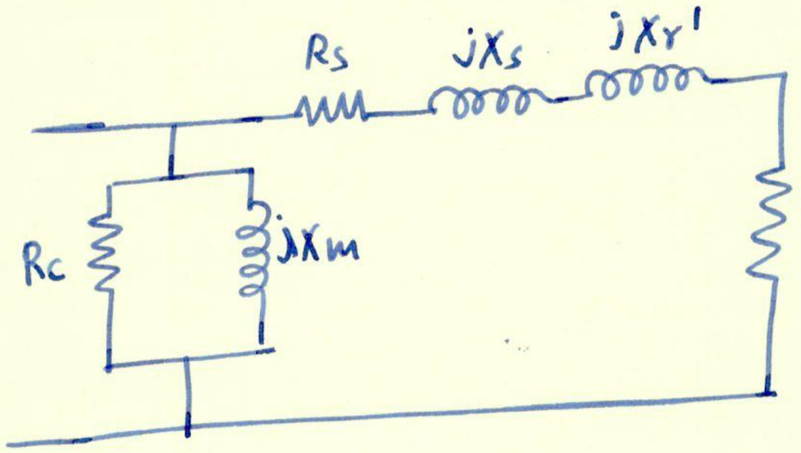
2

$$(I_{br/phase})^2 X_e = Q_{br/phase}$$

$$X_e = \frac{(38.3728)}{(2.77)^2} = 5.0010\ \Omega$$

$\Rightarrow X_s = X_r' \simeq 0.5 X_e \approx 2.5\ \Omega$

$$\boxed{\begin{array}{l} R_s = 1.2\ \Omega \quad R_r' = 1.3805 \\ X_s = X_r' = 2.5\ \Omega \\ R_c = 96.1422\ \Omega, \quad X_m = 128.04\ \Omega \end{array}}$$



$$W_{ms} = \frac{120 f}{P} \times \frac{2\pi}{60} = \frac{120 \times 60}{4} \times \frac{2\pi}{60} = 188.4955 \text{ rad/sec.}$$

$$\alpha = \frac{3 V_1^2 R_r'}{W_{ms}} = \frac{3\left(\frac{208}{\sqrt{3}}\right)^2 1.3805}{188.4955} = 316.856$$

$$A = R_s^2 + X_e^2 = (1.2)^2 + (5.001)^2 = 26.45$$

$$B = 2 R_s R_r' = 2(1.2)(1.3805) = 3.3132$$

$$C = R_r'^2 = 1.90578$$

$$T_{max(motoring)} = \frac{\alpha}{B + 2\sqrt{AC}} = \frac{316.856}{3.3132 + 2\sqrt{26.45 \times 1.90578}}$$

$$= \underline{18.0927 \text{ N-m.}}$$

$$s_{(max\,T,\ motoring)} = +\sqrt{\frac{C}{A}} = +\sqrt{\frac{1.90578}{26.45}}$$

$$= \underline{0.2684}$$

3

$$\text{Speed}\,(\max T)_{\text{motoring}} = (1 - 0.2684) \times 1800$$

$$= \underline{1316.8343 \text{ rpm}}$$

Ans 2 (A) 440 V, 3ϕ, 50 Hz, 6 pole, 945 rpm

Δ Connected

$R_s = 2\,\Omega$ $R_r' = 2\,\Omega$ $X_s = 3\,\Omega$ $X_r' = 4\,\Omega$

$X_m = 100\,\Omega$ $\quad W_{ms} = \frac{120 \times 50}{6} \times \frac{2\pi}{60} = 104.7197$ rad/sec

$T_L = K\,W_m$ $\rightarrow$ at rated voltage it runs at rated speed.

$$\alpha = \frac{3 V_1^2 R_r'}{W_{ms}} = \frac{3\,(440)^2\, 2}{104.7197} = 11092.4629$$

$$A = R_s^2 + X_e^2 = 2^2 + 7^2 = 53$$

$$B = 2 R_s R_r' = 2(2)(2) = 8$$

$$C = R_r'^2 = 4$$

$$s_{rated} = 1 - S_{rated} = 1 - \frac{945}{1000} = 0.055$$

$$T_{e\,(\text{rated } s)} = \frac{\alpha s}{A s^2 + B s + C} = \frac{(11092.4629)(0.055)}{53(0.055)^2 + 8(0.055) + 4}$$

$$= 13.061133 \text{ N-m.}$$

$$\Rightarrow \quad K\,W_{m(rated)} = T_L = T_e = 13.061133$$

$$\Rightarrow \quad K = \frac{13.061133}{945 \times \frac{2\pi}{60}} = 0.131983$$

(i) $W_m = 800$ rpm $= 83.7758$ rad/sec

$T_L = K\,W_m = 11.05698$ N-m.

$$s_{800rpm} = 1 - \frac{800}{1000} = 0.2$$

4

$$T_e = \frac{\alpha s}{A s^2 + B s + C} = T_L$$

$$\frac{\alpha_{(800rpm)}\, s_{800}}{A s_{800}^2 + B s_{800} + C} = 11.05698$$

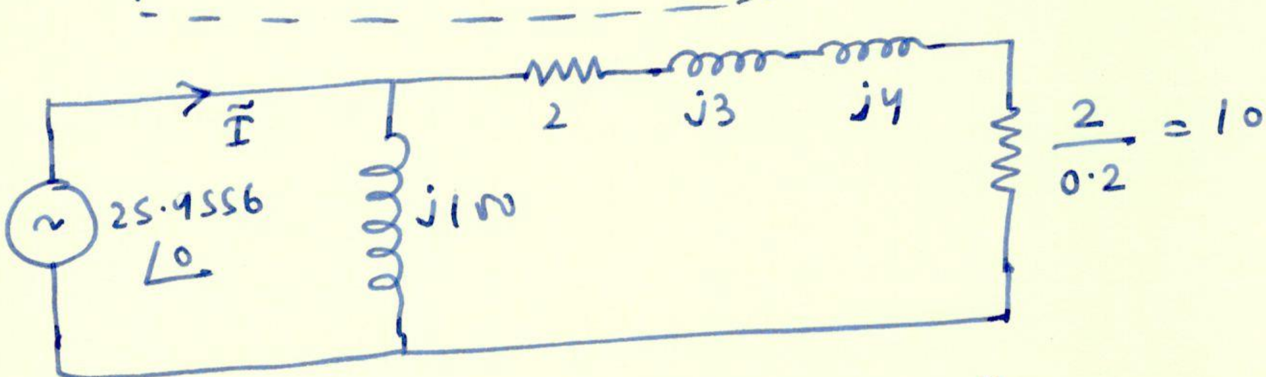
$$\alpha_{(800rpm)}\, 0.2 = (11.05698)(53(0.2)^2 + 8(0.2) + 4)$$

$$\alpha_{(800rpm)} = 38.6$$

$$\Rightarrow \frac{3 V_{(800)}^2 R_r'}{W_{ms}} = 38.6$$

$$\Rightarrow \frac{3 V_{(800)}^2 \times 2}{104.7197} = 38.6$$

$$\Rightarrow \boxed{V_{800} = 25.9556\ V}$$



$$\tilde{I} = \frac{25.9556\angle 0}{j100} + \frac{25.9556\angle 0^\circ}{12 + 7j}$$

$$= 2.01164 \angle -36.6553$$

$$I_{Line} = \sqrt{3} \times 2.01164 = 3.48426\ A.$$

$$T_{o\,(800rpm)} = T_{L\,(800rpm)} = 11.05698.$$

(ii) $V_L = 280 = V_{phase}$.

$$\frac{\alpha s}{As^2 + Bs + C} = K\omega_m = K\, S\, \omega_{ms}$$
$$= K(1-s)\,\omega_{ms}.$$

$$\alpha = \frac{3 \times (280)^2\, 2}{104.7197} = 4491.991478$$

$A = 53$, $B = 8$, $C = 4$, $K = 0.131983$.

$$\Rightarrow \frac{4491.991478\, s}{53s^2 + 8s + 4} = (0.131983)(104.7197)(1-s)$$

$$\Rightarrow 4491.991478\, s = 13.82122\,(1-s)(53s^2 + 8s + 4)$$

$$\Rightarrow 325.0068\, s = 53s^2 + 8s + 4 - 53s^3 - 8s^2 - 4s$$

$$\Rightarrow 325.0068\, s = -53s^3 + 45s^2 + 4s + 4$$

$$\Rightarrow 53s^3 - 45s^2 + 321.0068\, s - 4 = 0$$

$s = \underline{0.012482}$, $0.41828 + 2.423i$, $0.41828 - 2.423i$

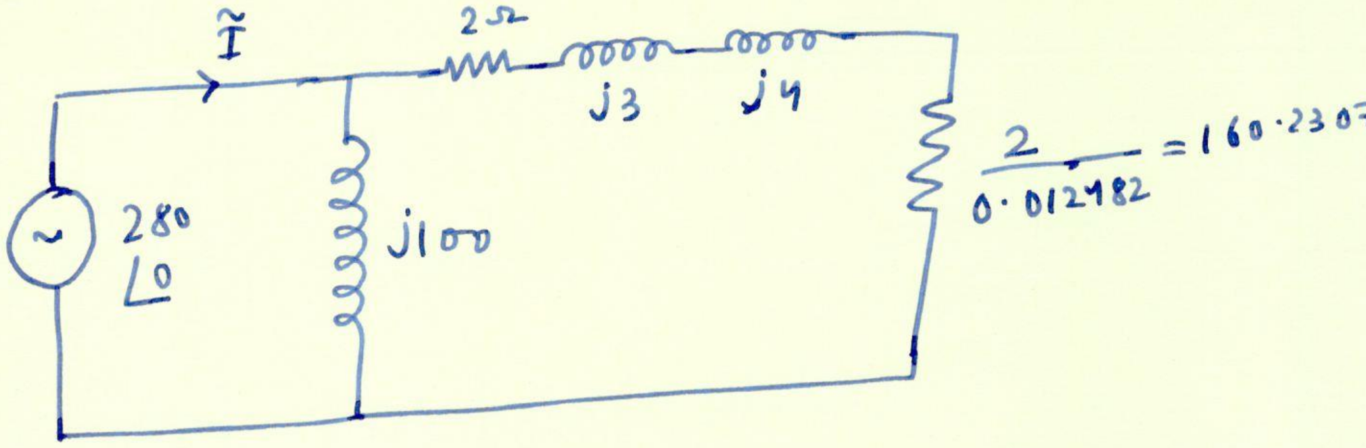
$\Rightarrow \underline{s = 0.012482} \Rightarrow \underline{S = 0.987518}$

$\omega_{m\,(280V)} = 0.987518 \times 1000 = 987.518$ rpm.

$T_e = T_L = K\omega_m = 0.131983 \times 987.518 \times \frac{2\pi}{60}$

$= \underline{13.6487 \text{ N-m}}$.

6



$$\tilde{I} = \frac{280\angle 0^\circ}{100j} + \frac{280\angle 0^\circ}{162.2307 + 7j}$$

$$= 3.351057 \angle -59.06366$$

$$\Rightarrow \underline{I_{LINE} = 3.351057\sqrt{3} = 5.8042\ A}$$

Ans 2(B) → Theory

Ans 3(A) 9 KVA, 208 V, 60 HZ, Y connected, 3Ø Synchronous generator

$Z_s = 0.1 + j3\ \Omega/\text{phase}$ $\quad R_f = 25\ \Omega$

$V_f = 120\text{V(dc)}$

$P_r = 800\ W$

per phase KVA $= \frac{9}{3} = 3$ KVA

per phase voltage $= \frac{208}{\sqrt{3}}$

$\Rightarrow$ rated load current $= \frac{3000}{208/\sqrt{3}} = 24.9815$ A.

(7)

$\tilde{V}_t = \frac{208}{\sqrt{3}} \angle 0^\circ = 120.088 \angle 0^\circ$

$\tilde{I}_a = 24.9815 \angle -25.8419^\circ$ (0.9 pf lag).

$\tilde{E}_a - \tilde{I}_a Z_s = \tilde{V}_t \Rightarrow \tilde{E}_a = \tilde{V}_t + \tilde{I}_a Z_s.$

$$\tilde{E}_a = 120.088 \angle 0^\circ + (24.9815 \angle -25.8419)(0.1 + j3)$$

$$= 168.611966 \angle 23.177^\circ$$

Approximate linearized slope $= \dfrac{45}{0.5}$

$$\Rightarrow \frac{45}{0.5} = \frac{168.611966 - 140}{I_f - 1}$$

$$\Rightarrow I_f - 1 = 0.31791$$

$$\Rightarrow I_f = 1.31791 \text{ A}$$

$$\frac{V_f}{R_f + R_{ex}} = I_f = 1.31791$$

$$\Rightarrow R_f + R_{ex} = 91.0532$$

$$\Rightarrow R_{ex} = 66.0532 \ \Omega$$

$$\% \ VR = \frac{E_a - V_t}{V_t} \times 100 = \frac{-\frac{208}{\sqrt{3}} + 168.611966}{\frac{208}{\sqrt{3}}}$$

8

$$= 40.407 \%$$

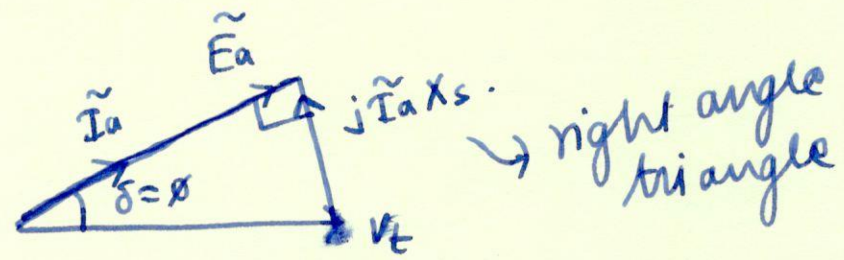
$$P_o = 3 V_t I_a \cos\phi = 3 \frac{208}{\sqrt{3}} (24.9815)(0.9) = 8099.999 \text{ W}$$

$$\eta = \frac{P_o}{P_o + \text{losses}} = \frac{8099.999}{8099.999 + (1.31791)^2 (91.0532) + 3(24.9815)^2(0.1) + 800}$$

$$= \frac{8099.999}{8099.999 + 158.1492 + 187.2226 + 800}$$

$$\%\eta = \frac{8099.999}{9245.3708} \times 100 = 87.6114\%$$

Ans 3(B)



for unity internal power factor $\delta = \phi$.

$$\boxed{\sin\phi = \frac{I_a X_s}{V_t}}$$ → governing dynamics for unity internal power factor

$$\sin\phi \leq 1$$

$$\Rightarrow \frac{I_a X_s}{V_t} \leq 1 \Rightarrow \boxed{I_a \leq \frac{V_t}{X_s}}$$

$$\boxed{I_a(\text{absolute maxima}) = \frac{V_t}{X_s}}$$

9

Ans 4.(A)

2200V, 60HZ, 3φ, Y connected 8 pole, synchronous motor

$$Z_s = 0.8 + j8\ \Omega$$

at no load $E_a = \frac{2200}{\sqrt{3}} = 1270.170592\ V$

$|\delta| = 22^0$ or $\delta = -22^0$

$$\tilde{V}_t = \frac{2200}{\sqrt{3}} \angle 0 = 1270.170592 \angle 0^\circ$$

$$\tilde{E}_a = \frac{2200}{\sqrt{3}} \angle -22^\circ = 1270.170592 \angle -22^\circ$$

$$\tilde{I}_a = \frac{\tilde{V}_t - \tilde{E}_a}{Z_s}$$

$$= 60.289 \angle \overset{\to \phi}{-5.2894}$$

$$P_{in} = 3\,Re(\tilde{V}_t \tilde{I}_a^*) = 3\, V_t I_a \cos\phi$$

$$= 3 \frac{2200}{\sqrt{3}} \times 60.289 \times \cos(5.2894)$$

$$= 228753.6931 \text{ W}$$

$$P_d = P_{in} - P_{cu}$$

$$= 228753.6931 - 3 \times (60.289)^2 \times 0.8$$

$$= 220030.2607 \text{ W}.$$

$$= 3\, Re(\tilde{E}_a \tilde{I}_a^*).$$

$$P_d = T_d \omega_{ms} \Rightarrow T_d = \frac{220030.2607}{\frac{120 \times 60}{8} \times \frac{2\pi}{60}}$$

$$T_d = \underline{2334.5935 \text{ N-m}}$$

* In this problem, Power developed cannot be evaluated by $\frac{3 E_a V_t}{X_s} \sin\delta$ as this is valid only when $R_a = 0$. ☺

10

Ans 4(B) 208 V, 60 Hz, 3ϕ, Y connected

Salient pole synchronous ~~generator~~ motor.

full load → 40 A and pf = 0.8 lead.

$X_d = 2.7\ \Omega \quad X_q = 1.7\ \Omega$

R_a = negligible and P_r = 5% of power developed.

$$\tilde{V}_t = \frac{208}{\sqrt{3}} \angle 0^\circ = 120.088 \angle 0^\circ$$

$$\tilde{V}_t - j\tilde{I}_d X_d - j\tilde{I}_q X_q = \tilde{E}_a$$

$$\tilde{V}_t - j\tilde{I}_a X_d - jX_q(\tilde{I}_a - \tilde{I}_d) = \tilde{E}_a$$

$$\tilde{V}_t - j\tilde{I}_d(X_d - X_q) - j\tilde{I}_a X_q = \tilde{E}_a$$

$$\Rightarrow \tilde{V}_t - j\tilde{I}_a X_q = \tilde{E}_a + j\tilde{I}_d(X_d - X_q)$$

$$= \tilde{E}_a'$$

→ Calculation of δ.

δ is angle of $\tilde{E}_a'$ also.

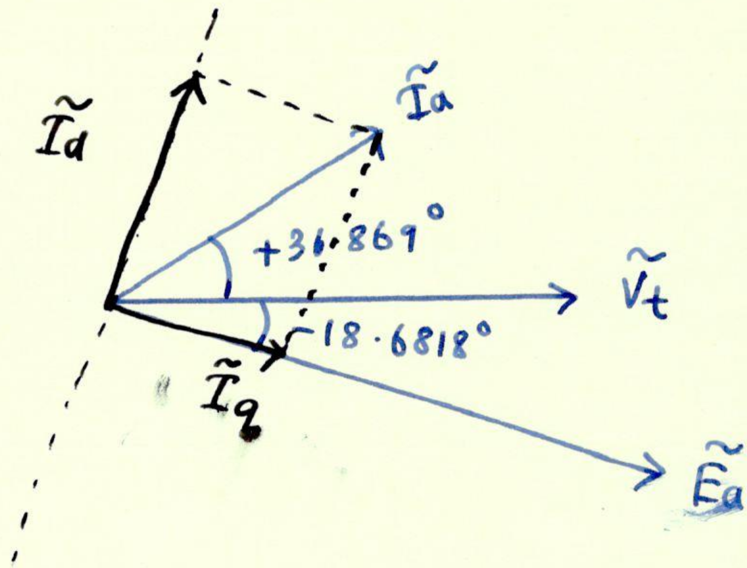
$$\Rightarrow \tilde{V}_t - j\tilde{I}_a X_q = 120.088 \angle 0 - j\, 40 \angle 36.869^\circ\, (1.7)$$

$$= 169.83552 \angle -18.6818^\circ$$

$$\Rightarrow \delta = -18.6818^\circ.$$

11

torque angle = δ = 18.6818°



$$\tilde{I}_q = 40\cos(36.869 + 18.6818)\angle -18.6818°$$

$$= 40\cos(55.5508)\angle -18.6818$$

$$= 22.627\angle -18.6818$$

$$\tilde{I}_d = 40\sin(55.5508)\angle 90° - 18.6818°$$

$$= 32.9851\angle 71.3182°$$

$$\tilde{E}_a = \tilde{V}_t - j\tilde{I}_d X_d - j\tilde{I}_q X_q$$

$$= 120.088 - j(2.7)\,32.9851\angle 71.3182°$$

$$- j(1.7)\,22.627\angle -18.6818°$$

$$\boxed{\tilde{E}_a = 202.82\angle -18.6818°}$$

12

$$P_{in} = 3V_t I_a \cos\phi = 3 \times 120.088 \times 40 \times 0.8$$

$$= 11528.448 = P_d \text{ (as no } R_a\text{)}$$

$$P_{synch} = \frac{3 E_a V_t}{X_d}\sin|\delta| = \frac{3(202.82)(120.088)}{2.7}\sin(18.6818)$$

$$\underline{P_{sync} = 8668.445366\ W}$$

$P_{salient} = P_d - P_{synch}$

$P_{salient} = 2860.002634$ W.

Ans 5(A) (i) 208 V, Y Connected, 3ϕ, Sync Motor

Takes 7.2 kW at 0.8 pf lead.

$X_s = 4\,\Omega \quad R_a \approx 0$

$3 V_t I_a \cos\phi = 7200$

$$I_a = \frac{7200}{3 \times \frac{208}{\sqrt{3}} \times 0.8} = 24.9815 \text{ A}.$$

as $R_a \approx 0 \quad P_{in} = P_d = -3\frac{E_a V_t}{X_s}\sin\delta.$

$\tilde{V_t} - j\tilde{I_a}X_s = \tilde{E_a}$

$$\Rightarrow \tilde{E_a} = \frac{208}{\sqrt{3}}\angle 0^\circ - j\, 24.9815\angle 36.869^\circ \times 4$$

$= 120.088 - (4j)(24.9815\angle 36.869^\circ)$

$= 196.9922\angle -23.942$

$$E_a' = \frac{196.9922}{2} = 98.4961$$

as the load on the motor is same hence P_d also must be same $= 7200$ W.

(13)

$$\Rightarrow -\frac{3 E_a' V_t}{X_s}\sin\delta' = 7200$$

$\Rightarrow \sin\delta' = -0.8116$

$\Rightarrow \delta' = -54.2544^\circ.$

$\Rightarrow$ New torque angle = 54.2544°.

$$\tilde{E}_a' = 98.4961 \angle -54.2544°.$$

$$\tilde{I}_a = \frac{\tilde{V}_t - \tilde{E}_a}{j X_s}$$

$$= \frac{120.088 \angle 0° - 98.4961 \angle -54.2544}{4 \angle 90°}$$

$$= 25.37572 \angle -38.04°$$

new armature current = 25.37572

at PF of $\cos(38.04) = 0.78758$ (lag)

(ii) $3 V_t I_a'' \cos\phi'' = 7200$

. $\cos\phi'' = 1$

$$\Rightarrow I_a'' = \frac{7200}{3 V_t} = \frac{7200}{3 \times 120.088}$$

$= 19.9853$ A

at UPF.

$$\tilde{E}_a = \tilde{V}_t - j \tilde{I}_a X_s.$$

$$= 120.088 \angle 0° - j4 \times (19.9853 \angle 0°).$$

14

$$= 144.2626 \angle -33.65124$$

New torque angle = 33.65124

new PF = unity

New armature current = 19.9853 A

Ans 5(B) Power developed by a salient pole synchronous motor can be expressed as

$$P_d = \underbrace{\frac{3 V_t}{X_d}}_{A} E_a \sin\delta + \underbrace{\frac{3 V_t^2}{2}\left(\frac{X_d - X_q}{X_d X_q}\right)}_{B} \sin 2\delta$$

When connected to infinite bus, V_t is constant

Also, as the motor is delivering constant power output $\Rightarrow$ P_d = constant {Neglecting R_a}

$$P_d = \text{constant} = A E_a \sin\delta + B \sin 2\delta$$

$E_a \to$ independent variable $\to X$

$\delta \to$ dependent variable $\to Y$

$$\Rightarrow \quad A X \sin Y + B \sin 2Y = \text{constant}$$

differentiating wrt X we get

15

$$A\left(\sin Y + X \cos Y \frac{dY}{dX}\right)$$

$$+ B\left(2 \cos 2Y \frac{dY}{dX}\right) = 0$$

$$\Rightarrow \quad \frac{dY}{dX}\left(A X \cos Y + 2B \cos 2Y\right) = -A \sin Y$$

$$\Rightarrow \quad \frac{dY}{dX} = \frac{-A \sin Y}{A X \cos Y + 2B \cos 2Y}$$

replacing X, Y with the original variables

$$\boxed{\frac{d\delta}{dE_a} = \frac{-A\sin\delta}{AE_a\cos\delta + 2B\cos 2\delta}}$$

$$A = \frac{3V_t}{X_d}\;;\quad B = \frac{3}{2}V_t^2\left(\frac{X_d - X_q}{X_d X_q}\right)$$

Above differential equation governs the steady state dynamics of torque angle/power angle with change in the excitation voltage for a synchronous m/c.