Generative AI for Math: Part I
MathPile: A Billion-Token-Scale Pretraining Corpus for Math

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https://github.com/GAIR-NLP/MathPile/

Abstract

High-quality, large-scale corpora are the cornerstone of building foundation models. In this work, we introduce MathPile, a diverse and high-quality math-centric corpus comprising about 9.5 billion tokens. Throughout its creation, we adhered to the principle of "*less is more*", firmly believing in the supremacy of data quality over quantity, even in the pretraining phase. Our meticulous data collection and processing efforts included a complex suite of preprocessing, prefiltering, language identification, cleaning, filtering, and deduplication, ensuring the high quality of our corpus. Furthermore, we performed data contamination detection on downstream benchmark test sets to eliminate duplicates. We hope our MathPilecan help to enhance the mathematical reasoning abilities of language models. We plan to open-source different versions of MathPilewith the scripts used for processing, to facilitate future developments in this field.

1 Introduction

Powerful conversational models such as ChatGPT (OpenAI, 2022) and Claude (Anthropic, 2023) are significantly transforming numerous products and aspects of daily life. A crucial factor in their success is the strength of the foundational language model. State-of-the-art foundation models are typically pretrained using massive, diverse and high-quality corpora, encompassing sources like Wikipedia, scientific papers, community forums, Github code, web pages and more (Gao et al., 2021; Together, 2023a). We expect a powerful foundational language model to possess comprehensive and balanced capabilities, including language understanding, commonsense reasoning, mathematical reasoning, language generation, and more (Bubeck et al., 2023).

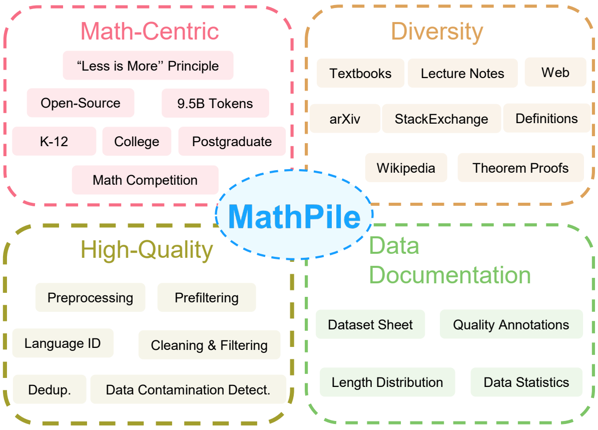

Figure 1: Key features of MathPile.

In this work, our concern centers on mathematical reasoning capabilities within foundational language models (Chern et al., 2023; Azerbayev et al., 2023b, *inter alia*), for which can potentially boost the application in education tools, automated problem solving, data analysis, code programming and so on, thereby improving user experience. To facilitate this, we are not directly building a model, but rather focusing on a more fundamental aspect: *creating a high-quality and diverse pre-training corpus tailored for the math domain*, namely MathPile. Specifically, our work is significantly different from the previous work in the following characteristics (See Table 1 for comparison):

Math-centric. Previous open-sourced pretraining corpora have typically focused on general domains, such as Pile (Gao et al., 2021), RedPajama (Together, 2023a) and Dolma (AllenAI, 2023). Others have concentrated on multilingual aspects or programming languages, such as ROOTS (Laurençon et al., 2022) and The Stack (Kocetkov et al., 2022), respectively. However, a notable absence in these offerings is a corpus specifically tailoring for mathematics. While

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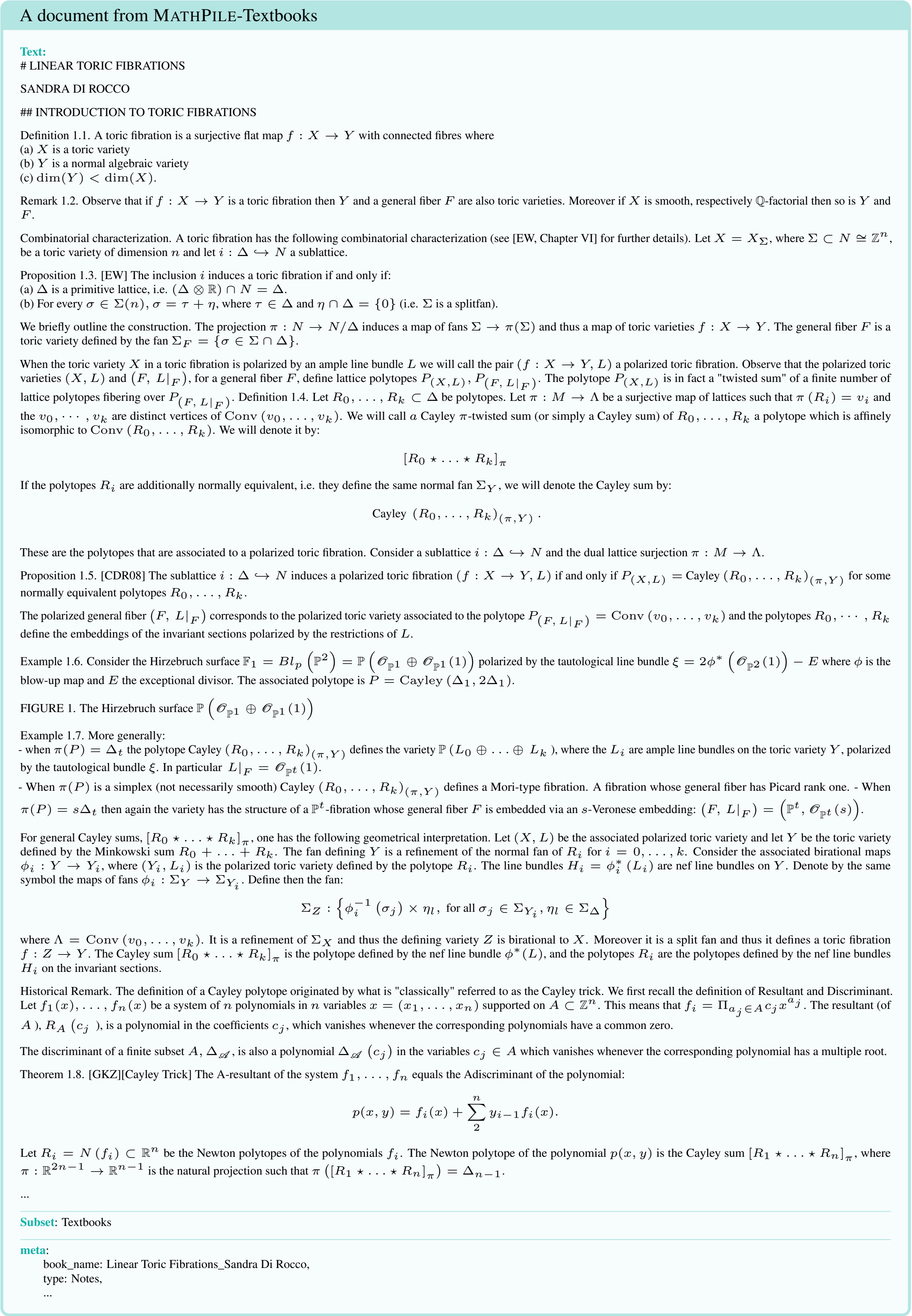
there exist some corpora designed for training or continually improving math-specific language models, such as Minerva's mathematical training dataset (Lewkowycz et al., 2022) and OpenAI's MathMix (Lightman et al., 2023), these are not open-sourced. Note that a recent work concurrent with ours, OpenWebMath (Paster et al., 2023), although math-centric, is solely sourced from web pages. We will discuss the comparison with it later. Recognizing this gap, our work aims to bridge the divide by developing an open-sourced mathematical corpus, democratizing access to high-quality mathematical data and enabling researchers and developers to advance the capabilities of language models in mathematical reasoning more effectively and inclusively.

Diversity. While Hendrycks et al. (2021b) introduced AMPS, a problem set ranging from elementary mathematics to multivariable calculus (K-12 level) for pre-training purposes, it lacks content at the college-level and more challenging competition-level mathematics, focusing instead on a supervised dataset rather than an extensive corpus. The ProofPile corpus, introduced by Azerbayev et al. (2023a), aims to improve autoformalization and formal proving capabilities in models, yet its scope is confined to formal proving, not covering the broader mathematical domain from K-12 to postgraduate level. Concurrently with our work, Paster et al. (2023) propose the OpenWebMath corpus, featuring a corpus composed of mathematical web pages. However, our corpus goes beyond web pages, integrating high-quality mathematics textbooks, lecture notes, scientific papers from arXiv in the field of mathematics, and carefully selected content from StackExchange, ProofWiki, and Wikipedia among others, which positions our corpus as a richer and more diverse mathematical resource for language models.

High-Quality. Recent studies have increasingly highlighted the detrimental effects of low-quality and repeated content in pretraining corpora on model training, as evidenced in various works (Allamanis, 2019; Luccioni and Viviano, 2021; Lee et al., 2022; Hernandez et al., 2022; Longpre et al., 2023). The importance of high-quality datasets has thus come to the fore. It has been shown that properly filtered and deduplicated web data can yield models as equally powerful as those trained on curated, high-quality corpora (Penedo et al., 2023). This similar practice has been recently adopted in several notable studies (Cerebras, 2023; AllenAI, 2023; Together, 2023b). A notable example is the 1.3 billion-parameter code-focused model pretrained on synthetically generated textbooks and filtered web pages, a project that broke existing scaling laws although did not open source its data (Gunasekar et al., 2023). It's important to emphasize that quality of the corpus is far more significant than its quantity. For instance, OpenAI's MathMix comprises only 1.5 billion tokens. In this work, we diligently adhere to the principle of *less is more*, as outlined in Zhou et al. (2023). To achieve a high-quality corpus, we have undertaken extensive preprocessing, prefiltering, cleaning, filtering, and deduplication efforts. We are committed to continually refining and optimizing this corpus, striving for excellence in every aspect to make a distinct contribution to the math domain.

Data Documentation. Auditing large-scale pretraining corpus, such as carefully documenting the characteristics of the data, intended uses, its information content, and any potential biases is of paramount importance (Bender and Friedman, 2018; Gebru et al., 2021; McMillan-Major et al., 2023). Despite growing advocacy for such practices, many pre-training corpus are released without detailed data documentation due to their large size (Mitchell et al., 2022). Recently, some works have audited certain publicly available pre-training datasets that previously lacked thorough documentation. These audits found that these corpus potentially contain useless content (e.g., hate speech, sexually explicit content) (Luccioni and Viviano, 2021; Kreutzer et al., 2022; Elazar et al., 2023), copyright-violating content (Bandy and Vincent, 2021), and the test sets for downstream tasks (Allamanis, 2019; Dodge et al., 2021). Adhering steadfastly to the principle of enhancing transparency in pretraining corpora for practitioners following previous efforts, we have provided a dataset sheet for our MathPile(see Table 5). Throughout our extensive data processing workflow, numerous documents were annotated for quality, such as language identification scores and the ratio of symbols to words (as exemplified in Figure 4). These quality annotations enable future users to apply their specific filters based on these scores. Additionally,



A document from MATHPILE-Textbooks

Text:
# LINEAR TORIC FIBRATIONS

SANDRA DI ROCCO

## INTRODUCTION TO TORIC FIBRATIONS

Definition 1.1. A toric fibration is a surjective flat map $f : X \rightarrow Y$ with connected fibres where
(a) X is a toric variety
(b) Y is a normal algebraic variety
(c) $\dim(Y) < \dim(X)$.

Remark 1.2. Observe that if $f : X \rightarrow Y$ is a toric fibration then Y and a general fiber F are also toric varieties. Moreover if X is smooth, respectively $\mathbb{Q}$-factorial then so is Y and F.

Combinatorial characterization. A toric fibration has the following combinatorial characterization (see [EW, Chapter VI] for further details). Let $X = X_\Sigma$, where $\Sigma \subset N \cong \mathbb{Z}^n$, be a toric variety of dimension n and let $i : \Delta \hookrightarrow N$ a sublattice.

Proposition 1.3. [EW] The inclusion i induces a toric fibration if and only if:
(a) Δ is a primitive lattice, i.e. $(\Delta \otimes \mathbb{R}) \cap N = \Delta$.
(b) For every $\sigma \in \Sigma(n)$, $\sigma = \tau + \eta$, where $\tau \in \Delta$ and $\eta \cap \Delta = \{0\}$ (i.e. Σ is a splitfan).

We briefly outline the construction. The projection $\pi : N \rightarrow N/\Delta$ induces a map of fans $\Sigma \rightarrow \pi(\Sigma)$ and thus a map of toric varieties $f : X \rightarrow Y$. The general fiber F is a toric variety defined by the fan $\Sigma_F = \{\sigma \in \Sigma \cap \Delta\}$.

When the toric variety X in a toric fibration is polarized by an ample line bundle L we will call the pair $(f : X \rightarrow Y, L)$ a polarized toric fibration. Observe that the polarized toric varieties (X, L) and $\left(F, L|_F\right)$, for a general fiber F, define lattice polytopes $P_{(X,L)}, P_{\left(F, L|_F\right)}$. The polytope $P_{(X,L)}$ is in fact a "twisted sum" of a finite number of lattice polytopes fibering over $P_{\left(F, L|_F\right)}$. Definition 1.4. Let $R_0, \ldots, R_k \subset \Delta$ be polytopes. Let $\pi : M \rightarrow \Lambda$ be a surjective map of lattices such that $\pi\left(R_i\right) = v_i$ and the $v_0, \cdots, v_k$ are distinct vertices of $\operatorname{Conv}\left(v_0, \ldots, v_k\right)$. We will call a Cayley π-twisted sum (or simply a Cayley sum) of $R_0, \ldots, R_k$ a polytope which is affinely isomorphic to $\operatorname{Conv}\left(R_0, \ldots, R_k\right)$. We will denote it by:

$$\left[R_0 \star \ldots \star R_k\right]_\pi$$

If the polytopes R_i are additionally normally equivalent, i.e. they define the same normal fan Σ_Y, we will denote the Cayley sum by:

$$\text{Cayley }\left(R_0, \ldots, R_k\right)_{(\pi, Y)} .$$

These are the polytopes that are associated to a polarized toric fibration. Consider a sublattice $i : \Delta \hookrightarrow N$ and the dual lattice surjection $\pi : M \rightarrow \Lambda$.

Proposition 1.5. [CDR08] The sublattice $i : \Delta \hookrightarrow N$ induces a polarized toric fibration $(f : X \rightarrow Y, L)$ if and only if $P_{(X,L)} =$ Cayley $\left(R_0, \ldots, R_k\right)_{(\pi, Y)}$ for some normally equivalent polytopes $R_0, \ldots, R_k$.

The polarized general fiber $\left(F, L|_F\right)$ corresponds to the polarized toric variety associated to the polytope $P_{\left(F, L|_F\right)} = \operatorname{Conv}\left(v_0, \ldots, v_k\right)$ and the polytopes $R_0, \cdots, R_k$ define the embeddings of the invariant sections polarized by the restrictions of L.

Example 1.6. Consider the Hirzebruch surface $\mathbb{F}_1 = Bl_p\left(\mathbb{P}^2\right) = \mathbb{P}\left(\mathscr{O}_{\mathbb{P}^1} \oplus \mathscr{O}_{\mathbb{P}^1}(1)\right)$ polarized by the tautological line bundle $\xi = 2\phi^*\left(\mathscr{O}_{\mathbb{P}^2}(1)\right) - E$ where ϕ is the blow-up map and E the exceptional divisor. The associated polytope is $P = \operatorname{Cayley}\left(\Delta_1, 2\Delta_1\right)$.

FIGURE 1. The Hirzebruch surface $\mathbb{P}\left(\mathscr{O}_{\mathbb{P}^1} \oplus \mathscr{O}_{\mathbb{P}^1}(1)\right)$

Example 1.7. More generally:
- when $\pi(P) = \Delta_t$ the polytope Cayley $\left(R_0, \ldots, R_k\right)_{(\pi, Y)}$ defines the variety $\mathbb{P}\left(L_0 \oplus \ldots \oplus L_k\right)$, where the L_i are ample line bundles on the toric variety Y, polarized by the tautological bundle ξ. In particular $L|_F = \mathscr{O}_{\mathbb{P}^t}(1)$.
- When $\pi(P)$ is a simplex (not necessarily smooth) Cayley $\left(R_0, \ldots, R_k\right)_{(\pi, Y)}$ defines a Mori-type fibration. A fibration whose general fiber has Picard rank one. - When $\pi(P) = s\Delta_t$ then again the variety has the structure of a $\mathbb{P}^t$-fibration whose general fiber F is embedded via an s-Veronese embedding: $\left(F, L|_F\right) = \left(\mathbb{P}^t, \mathscr{O}_{\mathbb{P}^t}(s)\right)$.

For general Cayley sums, $\left[R_0 \star \ldots \star R_k\right]_\pi$, one has the following geometrical interpretation. Let (X, L) be the associated polarized toric variety and let Y be the toric variety defined by the Minkowski sum $R_0 + \ldots + R_k$. The fan defining Y is a refinement of the normal fan of R_i for $i = 0, \ldots, k$. Consider the associated birational maps $\phi_i : Y \rightarrow Y_i$, where $\left(Y_i, L_i\right)$ is the polarized toric variety defined by the polytope R_i. The line bundles $H_i = \phi_i^*\left(L_i\right)$ are nef line bundles on Y. Denote by the same symbol the maps of fans $\phi_i : \Sigma_Y \rightarrow \Sigma_{Y_i}$. Define then the fan:

$$\Sigma_Z : \left\{\phi_i^{-1}\left(\sigma_j\right) \times \eta_l, \text{ for all } \sigma_j \in \Sigma_{Y_i}, \eta_l \in \Sigma_\Delta\right\}$$

where $\Lambda = \operatorname{Conv}\left(v_0, \ldots, v_k\right)$. It is a refinement of Σ_X and thus the defining variety Z is birational to X. Moreover it is a split fan and thus it defines a toric fibration $f : Z \rightarrow Y$. The Cayley sum $\left[R_0 \star \ldots \star R_k\right]_\pi$ is the polytope defined by the nef line bundle $\phi^*(L)$, and the polytopes R_i are the polytopes defined by the nef line bundles H_i on the invariant sections.

Historical Remark. The definition of a Cayley polytope originated by what is "classically" referred to as the Cayley trick. We first recall the definition of Resultant and Discriminant. Let $f_1(x), \ldots, f_n(x)$ be a system of n polynomials in n variables $x = \left(x_1, \ldots, x_n\right)$ supported on $A \subset \mathbb{Z}^n$. This means that $f_i = \Pi_{a_j \in A} c_j x^{a_j}$. The resultant (of A), $R_A\left(c_j\right.$), is a polynomial in the coefficients c_j, which vanishes whenever the corresponding polynomials have a common zero.

The discriminant of a finite subset $A, \Delta_{\mathscr{A}}$, is also a polynomial $\Delta_{\mathscr{A}}\left(c_j\right)$ in the variables $c_j \in A$ which vanishes whenever the corresponding polynomial has a multiple root.

Theorem 1.8. [GKZ][Cayley Trick] The A-resultant of the system $f_1, \ldots, f_n$ equals the Adiscriminant of the polynomial:

$$p(x, y) = f_i(x) + \sum_2^n y_{i-1} f_i(x) .$$

Let $R_i = N\left(f_i\right) \subset \mathbb{R}^n$ be the Newton polytopes of the polynomials f_i. The Newton polytope of the polynomial $p(x, y)$ is the Cayley sum $\left[R_1 \star \ldots \star R_n\right]_\pi$, where $\pi : \mathbb{R}^{2n-1} \rightarrow \mathbb{R}^{n-1}$ is the natural projection such that $\pi\left(\left[R_1 \star \ldots \star R_n\right]_\pi\right) = \Delta_{n-1}$.

...

Subset: Textbooks

meta:
book_name: Linear Toric Fibrations_Sandra Di Rocco,
type: Notes,
...

Figure 2: An example textbook document in MATHPILE

we have conducted extensive deduplication for this corpus and performed data contamination detection with downstream benchmark test sets, removing any duplicated samples identified (cf. § 3.4). Interestingly, we have also discovered a significant number of questions from downstream test sets in OpenWebMath (cf. § 3.4). This underscores the importance of meticulous data documentation. We plan to release different versions of MathPileto facilitate future use, further emphasizing the utility and adaptability of our work. See Appendix B for examples in MathPile.

In conclusion, we hope to facilitate the growth of the field of AI for mathematics by contributing this specialized, high-quality, diverse corpus focused on the mathematical domain while maintaining utmost transparency about the data for practitioners. Our work lays the groundwork for training more powerful mathematical problem-solving models in the future.

2 The Collection of Corpus

In order to construct MathPile, we gather data from a variety of sources, which also includes a component of manual collection.

2.1 Mathematical Textbooks

Textbooks are typically self-contained, encompassing mathematical concepts, exercises, and detailed solution steps. We believe that such resources are valuable for *educational purposes*, not only for human but also machine learning models. Some recent works have also corroborated this point, even though they didn't focus on the math domain, and their textbooks are not genuine but were synthesized from more advanced models (Gunasekar et al., 2023; Li et al., 2023).

To collect these genuine and high-quality textbooks, we began by conducting extensive manual searches across the internet, seeking open-source and freely accessible mathematics-related textbook websites. Afterwards, we proceeded to download these PDF files, resulting in a collection of 38 K-12 level textbooks, along with 369 college-level mathematics textbooks that cover a wide range of subjects including linear algebra, probability theory, calculus, and optimization. In addition to these textbooks, we also included 467 college course handouts and lecture notes, which tend to be more concise compared to full-length textbooks. Subsequently, we employed the Mathpix API[1] to parse the PDFs into markdown format. Then, we meticulously cleaned up extraneous elements such as parsed image URLs, preface sections, table of contents, acknowledge sections, index sections, and consecutive empty lines within the parsed content. After that, we arrived at a total of 874 documents.

We also refined high-quality mathematics-related synthetic textbooks from OpenPhi Project.[2] It is an open-source counterpart to the Phi work (Gunasekar et al., 2023). While the underlying model and generation process differ, the output encompasses a broad spectrum of subjects, extending beyond programming. To isolate mathematics-related documents, we employed a straightforward criterion: the presence of the symbol "$$", commonly associated with mathematical expressions. This approach yielded 3,889 documents from an initial pool of 124,493. As the volume of pre-training data escalates, the synthesis of high-quality data becomes increasingly crucial. More advanced filtering methods and mathematical corpora synthesis are left for future exploration.

2.2 Mathematical Papers from ArXiv

ArXiv offers a free distribution service and serves as an open-source archive housing millions of scientific papers. It also provides invaluable training data for numerous powerful language models (Touvron et al., 2023a; Together, 2023a, *inter alia*). In our endeavor to collect mathematical papers from ArXiv, we identify 50 sub-subjects spanning Mathematics, Computer Science, Statistics, Physics, Quantitative Finance and Economics. Our process involved filtering ArXiv's metadata[3] to focus on the chosen subjects (cf. Table 6), followed by accessing the source LaTex files (if available). We exclusively retained the LaTex files and consolidated multiple files based on their respective order as indicated by commands such as "include" and "input" within the main LaTex file of each paper. Subsequently, we undertook extensive transformations to enhance data clarity and consistency. Specifically, we

[1] https://mathpix.com/ocr

[2] https://huggingface.co/open-phi

[3] https://www.kaggle.com/datasets/Cornell-University/arxiv

Datasets	Open Source	Type	Target Domain	# Textbooks	Has Synth. Data	Data Contam. Detection	# Tokens	Source
Minerva	✗	Corpus	General Math	✗	✗	✓	38.5B	arXiv, Web
MathMix	✗	Corpus + PS	General Math	?	✓	✓	1.5B	?
ProofPile	✓	Corpus	Theorem Proving	7	✗	✗	8.3B	arXiv, Textbooks, Lib., StackExchange, ProofWiki, MATH
OpenWebMath	✓	Corpus	General Math	✗	✗	✗	14.7B	Web
DM-Mathematics	✓	PS	Math Competition	✗	✓	-	4.4B	Synthesis
AMPS	✓	PS	Math Competition	✗	✓	✗	0.7B	Khan Academy, Synthesis
MathPile(Ours)	✓	Corpus	General Math	3,979	✓	✓	9.5B	arXiv, Textbooks, StackExchange, Wikipedia, ProofWiki, Web

Table 1: The comparison of MathPilewith other mathematical Corpora. PS denotes the problem set type. For some corpora that are not open-sourced, details are unknown and comparisons are based only on information from corresponding papers, with unknowns indicated by "?". Note that token counts vary with different tokenizers; we primarily copy statistics from each dataset's technical report. For our corpus, we default to using the GPTNeoX-20B tokenizer (Black et al., 2022). DM-Mathematics was introduced in Saxton et al. (2019). We use "Minerva" to refer to the dataset adopted by Minerva. Note that ProofPile-2 (Azerbayev et al., 2023b), which includes OpenWebMath, RedPajama's arXiv subset (non-math-centric) and algebra code, is not included in this comparison.

1) removed comments in each paper;
2) reverted many macro commands (e.g., "`newcommand`") to their original forms;
3) omitted figure environments while retaining captions and figure labels;
4) excluded acknowledgements sections;
5) eliminated references in each paper;
6) condensed more than three consecutive empty lines to two;
7) replaced certain formatting commands like "`hfill`" and "`vspace`" with an empty line;
8) replaced the "`maketitle`" command in the main document body with the actual title (if available);
9) preserved only the content within the main body of the LaTex document.

Finally, we had a grand total of 347,945 meticulously cleaned LaTex documents (around 8.5 billion tokens), with each document corresponding to a single paper.

2.3 Mathematical Entries in Wikipedia

Wikipedia[4] is one the largest and most popular free online encyclopedias, offering information on a wide range of topics, including history, science, technology, culture, and more. This extensive knowledge has proven to be highly beneficial for numerous natural language processing tasks (Lewis et al., 2020, *inter alia*) and pretrained language models (Devlin et al., 2019; Touvron et al., 2023a, *inter alia*). To collect mathematical entries from Wikipedia, we downloaded the mathematics-focused (without pictures) dump of Wikipedia in English for the month of August 2023. We extracted the HTML documents from the dump using the library `libzim`,[5] resulting in approximately 106,900 documents. Subsequently, we converted these HTML documents into markdown format using the `html2text` library[6] while removing the hyperlinks following the practice of LLaMA (Touvron et al., 2023a). We retained the alternative text content but excluded image (often in SVG format) paths. Additionally, we eliminated extra newlines within paragraphs and condensed more than three consecutive empty lines to two using regular expressions. Further refinement involved the removal of boilerplate content at the bottom of the pages, typically denoted with phrases like "`This article is issued from Wikipedia. The text is ...`". In the end, our efforts yielded a collection of 106,881 mathematical Wikipedia entries, about 0.8 billion tokens.

2.4 Entries from ProofWiki

ProofWiki,[7] an online compendium of mathematical proofs, has been instrumental in advancing the fields of autoformalization and formal proof proving, as evidenced by NaturalProofs (Welleck et al., 2021) and ProofPile (Azerbayev et al., 2023a). We

[4] Wikipedia is licensed under CC BY-SA 4.0.

[5] `https://pypi.org/project/libzim/`

[6] `https://pypi.org/project/html2text/`

[7] ProofWiki is licensed under CC BY-SA 3.0.

sourced data from the ProofWiki dump dated April 9, 2022 (provided by the Internet Archive), mirroring the preprocessing approach employed by NaturalProofs, which was based on the version from November 12, 2020. Specifically, this involved leveraging the `BeautifulSoup`[8] to parse all wiki pages followed by the extraction of raw text content using the `wikitextparser` library.[9] This process yielded a substantial collection of mathematical content, totaling about 7.6 million tokens, comprising 10,328 definitions and 13,511 theorem-proof pairs. To facilitate better data organization, we formatted the definitions using the "`definition`" environment, and the theorem-proof pairs within the "`section`" environment with their respective titles serving as the section headings, in line with the format of ProofPile.

2.5 Mathematical Discussions on StackExchange

StackExchange,[10] renowned for its network of community-powered question-and-answering websites, spans a wide array of topics, each concentrated on a particular topic. Its high-quality data trove has significantly contributed to the development of various language models (Touvron et al., 2023a; Zhou et al., 2023, *inter alia*). In our study, we identify eleven sites within this network, including five dedicated to mathematics (such as Mathematics and MathOverflow) and six others in closely related fields like Physics (cf. Table 7).

Our data collection process began with downloading the site dumps from August 2023 (provided by the Internet Archive). In our data curation, we only retained the essential components in the posts, namely questions and answers (also associated meta information). To convert HTML documents to raw text, we utilized the `BeautifulSoup` library, coupled with a meticulous removal of invalid XML characters. We then systematically paired questions and their respective answers. Each question typically garners multiple responses, each with its own score and in some cases, an endorsement as the accepted answer by the questioner.

To ensure the high quality of our dataset, we leveraged two filtering score thresholds: a basic quality threshold set at 5, and a more stringent one at 10. Questions were filtered based on these thresholds, while answers were judged by the lesser of the threshold or the score of the accepted answers if one exists. Additionally, we also retained unanswered questions with at least a score of 10 as a reserve to facilitate future use.[11] Finally, our process has yielded a rich collection of data: 176,962 mathematics-intensive questions with 290,570 answers (filtered by the 5-score threshold), and 3,418 unanswered questions (10-score threshold). For the sites potentially related to mathematics, we have gathered 90,957 questions with 144,559 answers (5-score threshold). In total, the assembled questions and answers (5-score threshold), including filtered unanswered questions, amount to about 254 million tokens.

2.6 Mathematical Web Pages from Common Crawl

Common Crawl,[12] an invaluable resource that has been archiving a comprehensive and open repository of web crawl data since 2007, stands as a cornerstone for training many advanced language models, including GPT-3 (Brown et al., 2020) and LLaMA. In our endeavor to extract mathematical web pages, we focus on refining the web corpus from SlimPajama (Cerebras, 2023)-a cleaned and deduplicated counterpart of RedPajama, specifically targeting the SlimPajama-CommonCrawl and SlimPajama-C4 subsets.

Eschewing the common approach of using neutral network-based filtering, we opt for heuristic rule-based methods. Our procedure began with the creation of TF-IDF features, derived from our curated high-quality textbooks (cf. § 2.1). During this process, we removed the stop words, limited the features to a maximum of 10,000, and employed white space tokenization. Upon the observation of the resulting vocabulary, we identified 11 commonly used LaTex commands, integral to mathematical expressions. We utilize these commands as a basis for a hard match within each document. A document is classified as mathematical if it contains any of these commands along with the symbol "$$", typically indicative of a mathemati-

[8] https://pypi.org/project/beautifulsoup4/

[9] https://pypi.org/project/wikitextparser/

[10] ProofWiki is licensed under CC BY-SA 2.5 3.0 or 4.0, depending on the date of the content.

[11] We also provide an unfiltered version for future use.

[12] https://commoncrawl.org/terms-of-use/

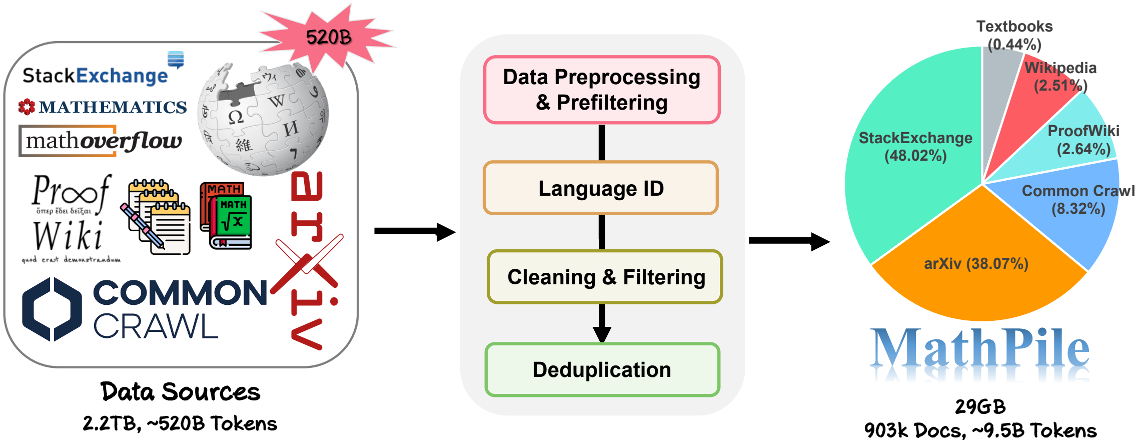


Figure 3: The creation process of MathPile. Beginning with data collection from diverse sources (about 520 B tokens), followed by our rigorous processing process, we obtain a math-centric corpus, encompassing 9.5 billion tokens. Note that we additionally perform data contamination detection on benchmark test sets (cf. § 3.4). We visualize the proportions of different components in MathPilebased on document counts per component (Right).

cal document. This rule-based approach, though simplistic, proved to be highly effective, especially given the vast size of the Common Crawl corpus . We also experimented with more intricate dense embedding-based methods to identify mathematical documents, but these resulted in poor recall.

Our efforts resulted in the compilation of a substantial collection of mathematical web pages: 4,307 documents from the SlimPajama-C4 training set and 72,137 documents from the SlimPajama-CommonCrawl training set, totaling approximately 633 million tokens. Note that there are possibilities for more efficient and effective methods to filter mathematical documents from the broader expanse of Common Crawl snapshots, a venture we aim to pursue in our future work.

3 Global Data Processing

While we have already conducted specific data preprocessing for each data source during the data collection process, we subsequently engage in three critical steps: **language identification**, **filtering**, and **deduplication**, to ensure the quality of the entire corpus, as shown in Figure 3.

3.1 Language Identification

To filter non-English documents, we utilized the fastText language identifier, which was trained on Wikipedia, Tatoeba, and SETimes (Joulin et al., 2017; Grave et al., 2018). A common practice is to classify a document as its respective language if the score exceeds 0.5, a threshold also employed by CCNet (Wenzek et al., 2020). However, during the application of this practice, we encountered a considerable number of false positives—cases where documents were erroneously filtered as non-English when, in fact, they were written in English but contained a substantial amount of mathematical symbols. We attribute this issue to the domain gap between the datasets used for fastText training (primarily wiki and news domain) and our mathematical content.

To more accurately filter out non-English documents, we set a customized score threshold for each data source to classify documents as English. Specifically, we set thresholds at 0.1 for Wikipedia and StackExchange, 0.3 for arXiv, 0.5 for Common Crawl. For ProofWiki and Textbooks, we opted not to employ this, as we had ensured during our manual collection process that all documents in these sources were written in English. As a result of this refinement process, approximately 8,400 documents were removed, amounting to a total 231 million tokens.

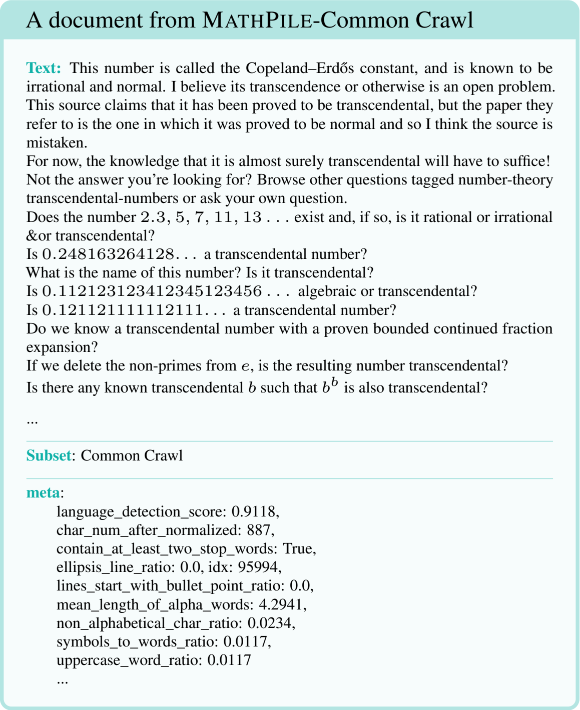
3.2 Data Cleaning and Filtering

Despite our meticulous and thorough data preprocessing efforts for each source during the corpus collection phase, we've noted that some documents, particularly from websites like Wikipedia and Common Crawl, are of insufficient quality used for language modeling. These documents might be too brief or include content that is either automatically generated or commonplace. While previous studies have introduced detailed methods for filtering pre-training corpora (Raffel et al., 2020; Rae et al., 2021; Longpre et al., 2023; Penedo et al., 2023; Cerebras, 2023), we found that these techniques are not entirely suitable for our math-focused corpus. Applying them as-is would lead to the exclusion of many valuable documents.

To address this issue, we developed a unique set of cleaning and filtering heuristic rules, specifically crafted for the mathematical domain and drawing from past studies. Specifically, we

1) detect lines containing “lorem ipsum” and filter them out if the resulting line is less than 5 characters;
2) detect lines containing “javascript” that also include “enable”, “disable” or “browser” and are under 200 characters, and filter them;
3) filter lines containing fewer than 10 words that include keywords like “Login”, “sign-in”, “read more...”, or “items in cart.”;
4) filter documents if the ratio of uppercase words exceeds 40%;
5) filter lines that end with “...” if they constitute more than 30% of the entire document;
6) filter documents if the ratio of non-alphabetic words surpasses 80%;
7) exclude documents with an average English word length outside the range of (3, 10);
8) discard documents that lack at least two common stop words such as “the”, “be” “to” “of” “and” “that” or “have”;
9) filter out documents if the ratio of ellipses (...) to words exceeds 0.5 (e.g., progress bars);
10) remove documents where 90% of lines start with bullet points;
11) filter documents including less than 200 characters after removing spaces and punctuation marks.

It’s these carefully formulated rules that have enabled us to curate a high-quality mathematical corpus. Furthermore, these rules have allowed us to assign quality annotations to each document (from Wikipedia and Common Crawl). These annotations offer future researchers and developers the flexibility to filter the data according to their criteria, tailoring it to their specific needs. We provide a cleaned example document with quality annotations, as shown in Figure 4. The process resulted in the filtration of approximately 1,100 documents, leading to the removal of 17 million tokens.



A document from MathPile-Common Crawl

Text: This number is called the Copeland–Erdős constant, and is known to be irrational and normal. I believe its transcendence or otherwise is an open problem. This source claims that it has been proved to be transcendental, but the paper they refer to is the one in which it was proved to be normal and so I think the source is mistaken.
For now, the knowledge that it is almost surely transcendental will have to suffice!
Not the answer you’re looking for? Browse other questions tagged number-theory transcendental-numbers or ask your own question.
Does the number $2.3, 5, 7, 11, 13 \ldots$ exist and, if so, is it rational or irrational &or transcendental?
Is $0.248163264128\ldots$ a transcendental number?
What is the name of this number? Is it transcendental?
Is $0.112123123412345123456\ldots$ algebraic or transcendental?
Is $0.121121111112111\ldots$ a transcendental number?
Do we know a transcendental number with a proven bounded continued fraction expansion?
If we delete the non-primes from e, is the resulting number transcendental?
Is there any known transcendental b such that b^b is also transcendental?

...

Subset: Common Crawl

meta:
language_detection_score: 0.9118,
char_num_after_normalized: 887,
contain_at_least_two_stop_words: True,
ellipsis_line_ratio: 0.0, idx: 95994,
lines_start_with_bullet_point_ratio: 0.0,
mean_length_of_alpha_words: 4.2941,
non_alphabetical_char_ratio: 0.0234,
symbols_to_words_ratio: 0.0117,
uppercase_word_ratio: 0.0117
...

Figure 4: An example document after cleaning and filtering with quality annotations

3.3 Data Deduplication

Given that our corpus originates from diverse sources, including the web and textbooks, it is inevitable that there will be repetitions both within and across these sources. Deduplication plays a crucial role in enhancing the training efficiency of the language model and reducing the memorization from the training data (Lee et al., 2022; Penedo et al., 2023). The challenge in deduplication lies in efficiently processing large-scale corpora to identify and eliminate not just exact duplicates but also near-duplicates. To this end, we employ the MinHash LSH algorithm built on the implementation of `text-dup` (Mou et al., 2023) and Lee et al. (2022). MinHash excels at efficiently estimating the similarity between sets at scale by transforming data into compact signatures using multiple hash functions (Broder, 1997). In the context of the text, the set corresponding to a document

In algebraic topology we often encounter chain complexes with extra multiplicative structure. For example, the cochain complex of a topological space has what is called the E_∞-algebra structure which comes from the cup product. *In this talk I present an idea for studying such chain complexes, E_∞ differential graded algebras (E_∞ DGAs), using stable homotopy theory. Namely, I discuss new equivalences between E_∞ DGAS that are defined using commutative ring spectra.* ring spectra are equivalent. *Quasi-isomorphic E_∞ DGAs are E_∞ topologically equivalent. However, the examples I am going to present show that the opposite is not true; there are E_∞ DGAs that are E_∞ topologically equivalent but not quasi-isomorphic. This says that between E_∞ DGAs, we have more equivalences than just the quasi-isomorphisms. I also discuss interaction of E_∞ topological equivalences with the Dyer-Lashof operations and cases where E_∞topological equivalences and quasi-isomorphisms agree.*	Özet : *In algebraic topology we often encounter chain complexes with extra multiplicative structure. For example, the cochain complex of a topological space has what is called the E_∞-algebra structure which comes from the cup product. In this talk I present an idea for studying such chain complexes, E_∞ differential graded algebras (E_∞ DGAs), using stable homotopy theory. Namely, I discuss new equivalences between E_∞ DGAS that are defined using commutative ring spectra*.We say E_∞ DGAs are E_∞ topologically equivalent when the corresponding commutative ring spectra are equivalent. *Quasi-isomorphic E_∞ DGAs are E_∞ topologically equivalent. However, the examples I am going to present show that the opposite is not true; there are E_∞ DGAs that are E_∞ topologically equivalent but not quasi-isomorphic. This says that between E_∞ DGAs, we have more equivalences than just the quasi-isomorphisms. I also discuss interaction of E_∞ topological equivalences with the Dyer-Lashof operations and cases where E_∞ topological equivalences and quasi-isomorphisms agree.*

Table 2: A near-duplication match found in CommonCrawl by MinHash LSH deduplication (in *italics*). See Appendix D for more examples.

consists of the collection of its n-grams. However, computing similarity for all possible pairs of such sets is time-consuming, hence the common practice is to employ a variant of MinHash known as Locality-Sensitive Hashing (LSH) (Gionis et al., 1999) which is efficient in identifying similar items by grouping documents with similar MinHash signatures into the same buckets, thereby reducing the number of pairwise comparisons needed. Specifically, our process involved splitting each document using whitespace and constructing 5-grams. We then applied the "`sha1`" hash function and configured the system with 450 buckets and 20 minhashes per bucket, resulting in a total of 9,000 minhashes for each document. This aligns with the setting outlined in RefinedWeb (Penedo et al., 2023).

During the deduplication process within each data source, we encountered a significant number of exact-match and near-duplicate documents. Specifically, we identified 304 duplicate documents in arXiv, 623 in Common Crawl, a notable 83,716 in Wikipedia, 783 in textbooks (mainly from synthetic textbooks), and 144 duplicate questions in Stack Exchange. Due to our standardization of the format from ProofWiki, despite detecting many near-duplicates through our deduplication process, we found that these were indeed different lemmas, proofs, or definitions (as we showcase in Table 10), thus, we ultimately did not deduplicate this source. Upon manual review, we made some interesting yet reasonable findings. For example, the significant duplication in Wikipedia was due to the collection of multiple historical versions of a document during the data collection. In StackExchange, duplication occurred because community members often posted similar questions on different sites (like Math and MathOverflow) to garner more responses (examples of which are provided in Table 13). We provide a near-duplicate example found in Common Crawl, as shown in 2 (See Table 8 to Table 13 for more examples from each data source). When deduplicating across different data sources, we hardly found any duplicate documents, except for one question from StackExchange that appeared in a document from Common Crawl. Therefore, we removed it. As a result of the deduplication process, about 714 million tokens were removed.

Note that we also experimented with using suffix arrays (Manber and Myers, 1993) to eliminate exact match sequences within documents. However, it tended to remove common phrases like "Questions: ". While it can effectively remove some templated content, it also disrupts the contextual integrity of our corpus. Consequently, we decided against employing this in order to preserve the context of our data.

3.4 Data Contamination Detection

As the scale of pre-training corpora expands, it is inevitable to encounter instances where examples from the evaluation set are found in the training set, a phenomenon known as *data contamination*. Generally, there are typically two types of data contamination, *input contamination*, where only the input of test examples appears in the training corpus, and *input-and-label contamination*, where both the inputs and their corresponding labels are present in the training corpus (Dodge et al., 2021). A common practice in past studies is to conduct the post-hoc data contamination analysis, utilizing n-gram overlap to assess the extent of contamination. For instance, GPT-2 performs an 8-gram approach (Radford et al., 2019), while GPT-3 (Brown et al., 2020) and FLAN (Wei et al., 2022) use 13-grams, and LLaMA-2 adopts more intricate skip-gram strategy (Touvron et al., 2023b). In this work, we advocate for the necessity of data contamination detection at the dataset creation stage itself, if feasible. Delaying this process until after training completion often results in irreversible damage (Kocetkov et al., 2022). Here, we utilize popular mathematical reasoning benchmarks, namely GSM8K (Cobbe et al., 2021), MATH (Hendrycks et al., 2021b), and MMLU-STEM (Hendrycks et al., 2021a), to detect data contamination.

Considering the variety of forms in which questions and answers might appear in pre-training corpora, we compiled the questions and answers from these benchmark tests into a set to serve as a reference for data contamination detection. It's important to note that for MMLU, which uses the multiple-choice format with typically short options, we considered only the questions. Intuitively, a math problem can have varied reasoning steps, making it relatively easier to detect the contamination of test questions in pre-training corpora. We employed line-level exact match detection for both our corpus and test sets, as the questions in these benchmarks are generally brief and often contained within a single line. Specifically, we split documents into lines, hashed each line using `MD5`, and took the first 64 bits along with the corresponding line to form a set. This procedure was also applied to the constructed reference test set collection. If a line from the test set, along with its corresponding hash code, is found in the training set's corresponding set, and the length of the line is over 50 characters,[13] we classify it as a leaked sample with an exact match.

Corpus	GSM8K	MATH	MMLU-STEM
Ours	-	23	2
OpenWebMath	-	195	65

Table 3: The occurrences of benchmark test sets in pre-training corpora detected by exact match. Note that some samples appear multiple times and the number only represents the lower bound of occurrences, as there may be some duplicates that have not been detected.

After conducting our detection process, we identified 23 questions from MATH and 2 from MMLU-STEM in our corpus (as shown in Table 3). These duplicates primarily appeared in StackExchange, Textbooks, and Common Crawl. Upon locating the questions within our corpus, we noted that no answers were provided following the questions. Table 14 and Table 15 showcase examples of these leaks from Textbooks and Common Crawl, along with their context. An interesting observation is that the leaks in Textbooks originated from AMC mathematics competition books, which coincidentally are also a source of questions in the MATH benchmark. We also applied this process to OpenWebMath, where we discovered many more duplicate questions from the MATH and MMLU test sets (also shown in Table 3), although many were duplicates. To illustrate, we provide some examples in Table 16. Interestingly, Azerbayev et al. (2023b) also report similar findings, albeit through a different detection method. This underscores the need for extra caution when creating pre-training corpora, as neglecting this can easily invalidate downstream benchmarks. Ultimately, we removed all detected exact matches from our corpus to mitigate data contamination issues. The resulting corpus is referred to as MATHPILE.

4 Data Analysis

Overview. As shown in Table 4, we present detailed statistical information for each component of MATHPILE, such as the number of documents and the count of tokens. Following our meticulous and comprehensive data collection and processing

[13] To avoid filtering out many common short phrases.

Components	Size (MB)	# Documents	# Tokens	max(# Tokens)	min (# Tokens)	ave (# Tokens)
Textbooks	644	3,979	187,194,060	1,634,015	256	47,046
Wikipedia	274	22,639	78,222,986	109,282	56	3,455
ProofWiki	23	23,839	7,608,526	6,762	25	319
CommonCrawl	2,560	75,142	615,371,126	367,558	57	8,189
StackExchange	1331	433,751	253,021,062	125,475	28	583
arXiv	24,576	343,830	8,324,324,917	4,156,454	20	24,211
Total	29,408	903,180	9,465,742,677	-	-	10,480

Table 4: The components and data statistics of MATHPILE.

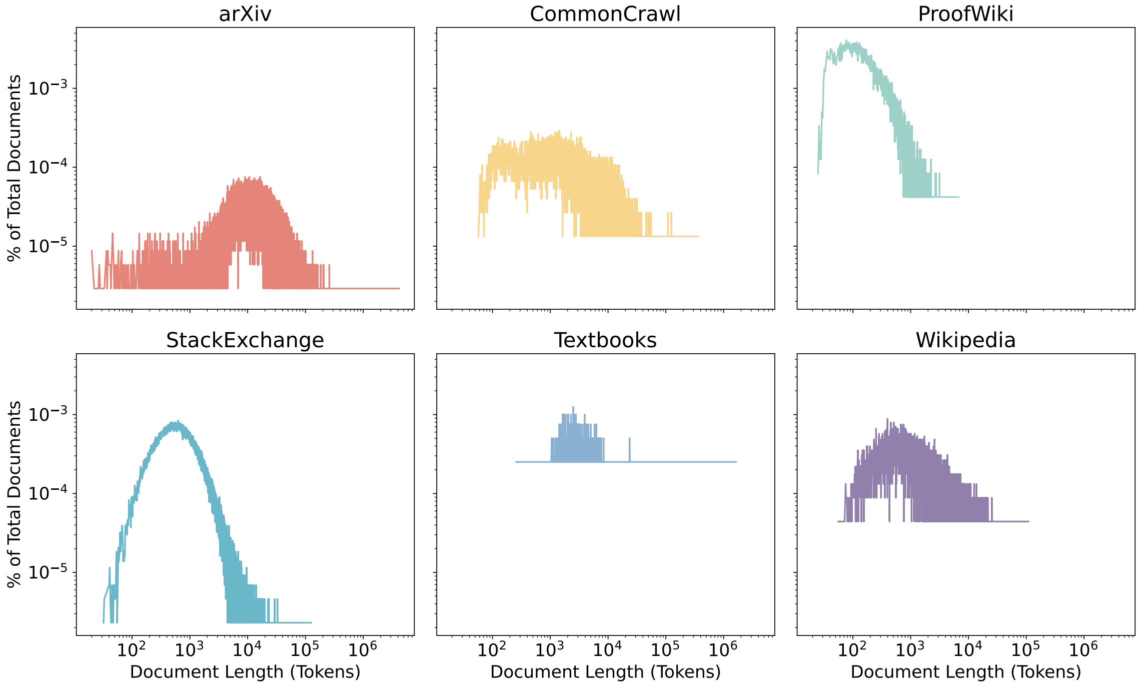


Figure 5: Document length distribution for different sources in MATHPILE(log-scale).

process, we obtain 29GB of high-quality and diverse math-centric corpus, encompassing around 9.5 billion tokens, from an initial volume of 2.2TB of raw data (cf. Figure 3). Compositionally, arXiv constitutes the largest portion of MATHPILE, while the Textbooks represent the smallest share, yet they are of exceptionally high quality.

The Length Distribution of Documents. We analyze the document length (in terms of token numbers) and their respective proportions from each source within MATHPILE, which is visualized in Figure 5. Intuitively, if the data from each source contains a higher amount of near-duplicates or machine-generated content, the distribution of documents of similar lengths becomes more prevalent, leading to a less smooth distribution curve. Figure 5 shows that, thanks to our thorough and rigorous processing, the document length distribution in MATHPILEis relatively smooth across different sources. Note that ProofWiki, due to its fixed format of definitions, lemmas, and proofs, naturally contains shorter content, resulting in a distribution with many similar lengths. We can also observe that, on average, the documents from arXiv and Textbooks tend to be lengthier, while those from ProofWiki and StackExchange are generally shorter.

5 Related Work

Pre-training Corpora for Language Models. In the field of language modeling, early models such as GPT (Radford et al., 2018) and BERT (Devlin et al., 2019) are primarily pre-trained on resources like Books (Zhu et al., 2015) and Wikipedia. Subsequent developments, including GPT-2 (Radford et al., 2019) and T5 (Raffel et al., 2020), expand the scope of training corpus to encompass web pages from sources like Reddit (resulting in WebText) and Common Crawl (resulting in C4). GPT-3 (Brown et al., 2020) marks a significant leap, enlarging its pre-training corpus to 300 billion tokens, utilizing Common Crawl, WebText, Books, and Wikipedia. Gao et al. (2021) introduce the Pile, a comprehensive collection of 22 diverse and high-quality datasets, specifically designed for the pre-training of large-scale language models. The Gopher project (Rae et al., 2021), although not open-sourced, compiles an extensive corpus of approximately 10.5TB, including web pages, books, new articles, and code. Similarly, the PaLM work (Chowdhery et al., 2023) develop a high-quality corpus of 780 billion tokens, spanning filtered web pages, books, Wikipedia, news, code, and social media conversations, yet it remained closed-source. On the other hand, BLOOM (Scao et al., 2022), an open-sourced multilingual models, is pre-trained on the ROOTS dataset (Laurençon et al., 2022), which aggregates content from hundreds of sources across 59 languages. Kocetkov et al. (2022) build The Stack, a 3.1 TB code dataset in 30 programming languages. LLaMA's training involved a diverse mixture of data, including sources like arXiv and StackExchange, in addition to the aforementioned sources (Touvron et al., 2023a). However, LLaMA did not release its corpus, in contrast to RedPajama, which did offer an open-source version (Together, 2023a). SlimPajama further enhances RedPajama by conducting an extensive deduplication process (Cerebras, 2023). RefinedWeb demonstrates the potential for web-only corpora to achieve performance comparable to well-curated corpus, such as Pile (Penedo et al., 2023). Recently, as the competition intensifies in the field of large language models, many more powerful models, such as GPT-4 (OpenAI, 2023), Mistral-7B (Jiang et al., 2023) and the lastest Gemini (Team et al., 2023), in addition to not open-sourcing their data, are also refraining from disclosing detailed information about their corpus in their technical reports. For the open-source community, constructing high-quality and diverse pre-training corpora is a crucial factor in bridging the performance gap with closed-source models. This is precisely the contribution we aim to make with our work.

Pre-training Benchmarks and Corpora for Mathematical Reasoning. Teaching models to possess mathematical reasoning abilities akin to humans is considered a vital aspect of achieving advanced artificial intelligence. This challenge has garnered widespread attention from both the machine learning and natural language communities. To better gauge the mathematical reasoning capabilities of models, numerous benchmark datasets have been introduced. such as AQuA (Ling et al., 2017), SVAMP (Patel et al., 2021), DM-Mathematics (Saxton et al., 2019), GSM8K (Cobbe et al., 2021) and MATH (Hendrycks et al., 2021b), to name a few. These datasets feature problems ranging from basic arithmetic operations to competition-level mathematics questions, encompassing a wide spectrum of difficulty. In addition, some benchmarks focus on the theorem-proving abilities, such as NaturalProofs (Welleck et al., 2021). The STEM subset of MMLU (Hendrycks et al., 2021a) concentrates on evaluating multi-task understanding in science, technology, engineering, and mathematics. To enhance the mathematical reasoning capabilities of language models, some pre-training corpora have been proposed. AMPS (Hendrycks et al., 2021b), although a large-scale synthetic exercise set, mainly targets problem-solving at the difficulty level of the MATH dataset. ProofPile focuses on mathematical theorem proving (Azerbayev et al., 2023a). Concurrently with our work, OpenWebMath (Paster et al., 2023) constructs a large-scale mathematical corpus, but is solely sourced from web pages. On the other hand, Google's corpus used for training Minerva (Lewkowycz et al., 2022) and the OpenAI's MathMix corpus (Lightman et al., 2023) are not open-sourced. Our work is dedicated to bridging this gap by constructing a high-quality mathematical corpus from diverse sources.

6 Conclusion

In this work, we present MathPile, a specialized corpus centered around mathematics, characterized by its diversity and high quality. Throughout its development, we meticulously source and gather data, applying a rigorous and math-specific pipeline. This pipeline encompasses various stages such as preprocessing, prefiltering, language identification, cleaning and filtering, and deduplication, all aimed at maintaining the high quality of the corpus. Note that we also conduct data contamination detection to remove duplicates from popular mathematical reasoning benchmark test sets, which is crucial for ensuring the integrity and effectiveness of these benchmarks in evaluating language models. However, this is an aspect often overlooked in other similar works. We hope that our MATHPILEcan be utilized, either independently or in collaboration with other corpora, to enhance the mathematical reasoning capabilities of language models, thereby fostering widespread applications.

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Limitations

The decisions made during the data collection and processing phases might not always be optimal, as verifying their effectiveness through low-cost computational methods is not feasible without training at each step. Therefore, our approach has been to draw upon the work of predecessors and, building on that foundation, cautiously navigate the specific challenges within the mathematical domain. This is because the practices of previous work may not always be entirely suitable for our math-focused scenario. Despite our significant efforts, the resulting corpus may not always be of the highest quality, especially documents sourced from the web, where a few low-quality documents might still persist.

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A MATHPILEDatasheet

MOTIVATION	
For what purpose was the dataset created?	Developed in a context where datasets like Google's Minerva and OpenAI's MathMix are not open-sourced, MATHPILEaims to counter this trend by enriching the open-source community and enhancing mathematical language modeling with its (relatively) large-scale, math-centric, diverse, high-quality dataset. It can be used on its own or cooperated with general domain corpora like books, and Github code, to improve the reasoning abilities of language models.
Who created the dataset and on behalf of which entity?	MATHPILEwas created by the authors of this work.
Who funded the creation of the dataset?	The creation of MATHPILEwas funded by GAIR Lab, SJTU.
Any other comment?	None.
COMPOSITION	
What do the instances that comprise the dataset represent?	MATHPILEis comprised of text-only documents, encompassing a broad range of sources. These include academic papers from arXiv, educational materials such as textbooks and lecture notes, definitions, theorems and their proofs, informative articles from Wikipedia, interactive Q&A content from StackExchange community users, and webpages sourced from Common Crawl. All these instances are math-focused.
How many instances are there in total?	MATHPILEcontains about 903 thousand of documents, or around 9.5 billion tokens.
Does the dataset contain all possible instances or is it a sample (not necessarily random) of instances from a larger set?	MATHPILEis curated from a diverse array of sources, including arXiv, Textbooks, Wikipedia, StackExchange, ProofWiki, and Common Crawl. However, it doesn't encompass all instances from these sources. We have implemented a rigorous data processing pipeline, which involves steps like preprocessing, prefiltering, language identification, cleaning, filtering, and deduplication. This meticulous approach is taken to guarantee the high quality of the content within MATHPILE.
What data does each instance consist of?	Each instance in MATHPILEis a text-only document, uniquely identified by its source, labeled under `Subset`. These instances are enriched with metadata, such as the score from language identification, the ratio of symbols to words, and their respective file paths. Note that instances from the StackExchange are composed of a question and its accompanying answers, each with their own set of meta data, including community users. To illustrate them, we provide specific examples for each source, ranging from Figure 6 to Figure 12.

Is there a label or target associated with each instance?	No.
Is any information missing from individual instances?	No.
Are relationships between individual instances made explicit?	No.
Are there recommended data splits?	No.
Are there any errors, sources of noise, or redundancies in the dataset?	Despite our rigorous efforts in cleaning, filtering out low-quality content, and deduplicating documents, it's important to acknowledge that a small fraction of documents in MathPilemight still fall short of our quality standards, particularly those sourced from web pages.
Is the dataset self-contained, or does it link to or otherwise rely on external resources?	Yes, MathPileis self-contained.
Does the dataset contain data that might be considered confidential?	No.
Does the dataset contain data that, if viewed directly, might be offensive, insulting, threatening, or might otherwise cause anxiety?	We do not expect offensive content despite our significant efforts in cleaning and filtering. But, we can not fully guarantee this.
Collection	
How was the data associated with each instance acquired?	Our data is primarily sourced from the arXiv website and the Internet Archive. The CommonCrawl data originates from SlimPajama. The textbooks included are manually collected, with quality checks performed on publicly available textbooks from various internet sources.
What mechanisms or procedures were used to collect the data?	Refer to § 2 for details on how they collect data.
If the dataset is a sample from a larger set, what was the sampling strategy?	We strive to use the most recent data dumps available and then selectively choose high-quality documents that are closely related to mathematics.
Who was involved in the data collection process and how were they compensated?	Authors from this paper were involved in collecting it and processing it.
Over what timeframe was the data collected?	MathPileencompasses documents created between 2007 and August 2023. Note that some documents and textbooks included may be created in the previous century.
Were any ethical review processes conducted?	No.
Preprocessing	

Was any preprocessing/cleaning/labeling of the data done?	Yes, during our data collection phase, we conducted extensive filtering and cleansing procedures, detailed in § 2. After the completion of data collection, we conducted further steps including language identification, additional cleaning and filtering, deduplication, and leakage detection in benchmark datasets. Subsequently, we removed any contaminated examples identified through this process. See § 3 for details.
Was the "raw" data saved in addition to the preprocessed/cleaned/labeled data?	Yes.
Is the software that was used to preprocess/clean/label the data available?	Yes, we will open-source the corresponding scripts.
USES	
Has the dataset been used for any tasks already?	Yes, this data has been used to develop mathematical language models.
Is there a repository that links to any or all papers or systems that use the dataset?	No.
What (other) tasks could the dataset be used for?	MathPilewas developed to enhance language modeling, offering significant benefits for a variety of mathematical reasoning tasks.
Is there anything about the composition of the dataset or the way it was collected and preprocessed/-cleaned/labeled that might impact future uses?	Our cleaning and filtering processes, while thorough, may not be entirely optimal, potentially leading to the exclusion of some valuable documents. Additionally, MathPileis specifically tailored for English, which limits its applicability in multilingual contexts.
Are there tasks for which the dataset should not be used?	Any tasks which may considered irresponsible or harmful.
DISTRIBUTION	
Will the dataset be distributed to third parties outside of the entity on behalf of which the dataset was created?	Yes, MathPilewill be available on the Huggingface Hub.
How will the dataset will be distributed?	MathPilewill be made available through the HuggingFace Hub.
When will the dataset be distributed?	The MathPilewill be available after this paper is made public.
Will the dataset be distributed under a copyright or other intellectual property (IP) license, and/or under applicable terms of use (ToU)?	If the source data of MathPileis governed by a license more restrictive than CC BY-NC-SA 4.0, MathPileadheres to that stricter licensing. In all other cases, it operates under the CC BY-NC-SA 4.0 license.
Have any third parties imposed IP-based or other restrictions on the data associated with the instances?	Not to our knowledge.

Do any export controls or other regulatory restrictions apply to the dataset or to individual instances?	Not to our knowledge.
MAINTENANCE	
Who will be supporting/hosting/maintaining the dataset?	MATHPILEwill be hosted on the HuggingFace Hub.
How can the owner/curator/manager of the dataset be contacted?	`stefanpengfei@gmail.com` `zzwang.nlp@gmail.com`
Is there an erratum?	No.
Will the dataset be updated?	Yes, it is currently a work in progress and updates are ongoing.
If others want to extend/augment/build on/contribute to the dataset, is there a mechanism for them to do so?	No.

Table 5: Datasheet for MATHPILE, following the framework introduced by Gebru et al. (2021).

B Examples of MathPile

We provide some illustrative examples from each source in MathPile, as shown in Figure 6 to Figure 12.

A document from MathPile-CommonCrawl

Text:
Are there optimizers where it is possible to specify ordinal ranking of parameters?
Assume that f is smooth (n-th order differentiable in each of the parameters).
An approach I often use when applying unconstrained optimisation algorithms to constrained problems is to transform the parameter space such that the constraints cannot be violated. Of course this results in $\theta_1^* \geq \theta_2^* \geq \theta_3^*$ which isn't quite what you asked for. To get a strict ranking you'll need to bump $x_1 - x_2^2$ and $x_1 - x_2^2 - x_3^2$ down at the last digit of precision.
thus spake a.k.thus spake a.k.
These variants of your constraints are linear, so provided that your function f is well-behaved (smooth, easy to calculate, easy to compute derivatives, derivatives are well-conditioned, etc.), any constrained optimization solver should be able to solve your problem without issue.
Not the answer you're looking for? Browse other questions tagged optimization constrained-optimization or ask your own question.
Does the amount of correlation of model parameters matter for nonlinear optimizers?
Optimization of a blackbox function with an equality constraint?

...

Subset: CommonCrawl

meta:
language_detection_score: 0.8670,
char_num_after_normalized: 926,
contain_at_least_two_stop_words: True,
ellipsis_line_ratio: 0.0,
idx: 383668,
lines_start_with_bullet_point_ratio: 0.0,
mean_length_of_alpha_words: 5.0870,
non_alphabetical_char_ratio: 0.0,
symbols_to_words_ratio: 0.0,
uppercase_word_ratio: 0.0060,
...

Figure 6: An example Common Crawl document in MathPile

C Details for Corpus Collection

The subjects from which we collected papers on arXiv are listed in Table 6. The specific StackExchange sites from which we gathered data are listed in Table 7.

Subjects
math.AG, math.AT, math.AP, math.CT, math.CA, math.CO, math.AC, math.CV, math.DG, math.DS, math.FA, math.GM, math.GN, math.GT, math.GR, math.HO, math.IT, math.KT, math.LO, math.MP, math.MG, math.NT, math.NA, math.OA, math.OC, math.PR, math.QA, math.RT, math.RA, math.SP, math.ST, math.SG, math-ph, quant-ph, cs.CC, cs.CG, cs.DM, cs.DS, cs.FL, cs.GT, cs.LG, cs.NA, cs.LO, q-fin.MF, stat.CO, stat.ML, stat.ME, stat.OT, stat.TH, econ.TH

Table 6: The subject list during collecting corpus from arXiv.

Sites sourced from StackExchange
math.stackexchange.com, mathoverflow.net, mathematica.stackexchange.com, matheducators.stackexchange.com, hsm.stackexchange.com, physics.stackexchange.com, proofassistants.stackexchange.com, tex.stackexchange.com, datascience.stackexchange, cstheory.stackexchange.com, cs.stackexchange.com

Table 7: The site list during collecting corpus from StackExchange.

D Examples of Duplicates Encountered in the Deduplication Process

We provide some illustrative examples of duplicates from each source in the deduplication process, as shown in Table 8 to Table 13.

We also provide examples of downstream task benchmarks (i.e., MATH and MMLU-STEM) leaks identified during our data contamination detection process for our corpus (as shown in Table 14 and Table 15) and OpenWebMath 16 (as shown in Table 16).

A document from MathPile-Wikipedia

Text:
# Inner Automorphism

In abstract algebra, an **inner automorphism** is an automorphism of a group, ring, or algebra given by the conjugation action of a fixed element, called the *conjugating element*. They can be realized via simple operations from within the group itself, hence the adjective "inner". These inner automorphisms form a subgroup of the automorphism group, and the quotient of the automorphism group by this subgroup is defined as the outer automorphism group.

## Definition

If G is a group and g is an element of G (alternatively, if G is a ring, and g is a unit), then the function

$$\varphi_g : G \to G$$
$$\varphi_g(x) := g^{-1}xg$$

is called **(right) conjugation by** g (see also conjugacy class). This function is an endomorphism of G: for all $x_1, x_2 \in G$,

$$\varphi_g(x_1x_2) = g^{-1}x_1x_2g = (g^{-1}x_1g)(g^{-1}x_2g) = \varphi_g(x_1)\varphi_g(x_2),$$

where the second equality is given by the insertion of the identity between x_1 and x_2. Furthermore, it has a left and right inverse, namely $\varphi_{g^{-1}}$. Thus, φ_g is bijective, and so an isomorphism of G with itself, i.e., an automorphism. An **inner automorphism** is any automorphism that arises from conjugation.[1]
When discussing right conjugation, the expression $g^{-1}xg$ is often denoted exponentially by x^g. This notation is used because composition of conjugations satisfies the identity: $(x^{g_1})^{g_2} = x^{g_1g_2}$ for all $g_1, g_2 \in G$. This shows that right conjugation gives a right action of G on itself.

### Inner and Outer Automorphism Groups

The composition of two inner automorphisms is again an inner automorphism, and with this operation, the collection of all inner automorphisms of G is a group, the inner automorphism group of G denoted Inn(G).
Inn(G) is a normal subgroup of the full automorphism group Aut(G) of G. The outer automorphism group, Out(G), is the quotient group

$$\text{Out}(G) = \frac{\text{Aut}(G)}{\text{Inn}(G)}.$$

The outer automorphism group measures, in a sense, how many automorphisms of G are not inner. Every non-inner automorphism yields a non-trivial element of Out(G), but different non-inner automorphisms may yield the same element of Out(G).
Saying that conjugation of x by a leaves x unchanged is equivalent to saying that a and x commute:

$$a^{-1}xa = x \iff xa = ax.$$

Therefore, the existence and number of inner automorphisms that are not the identity mapping is a kind of measure of the failure of the commutative law in the group (or ring).
An automorphism of a group G is inner if and only if it extends to every group containing G.[2]

...

Subset: Wikipedia

meta:
language_detection_score: 0.7236,
char_num_after_normalized: 5794,
contain_at_least_two_stop_words: True,
ellipsis_line_ratio: 0.0,
lines_start_with_bullet_point_ratio: 0.0,
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mimetype: text/html,
page_index: 48171,
page_path: A/Inner_automorphism,
page_title: Inner automorphism,
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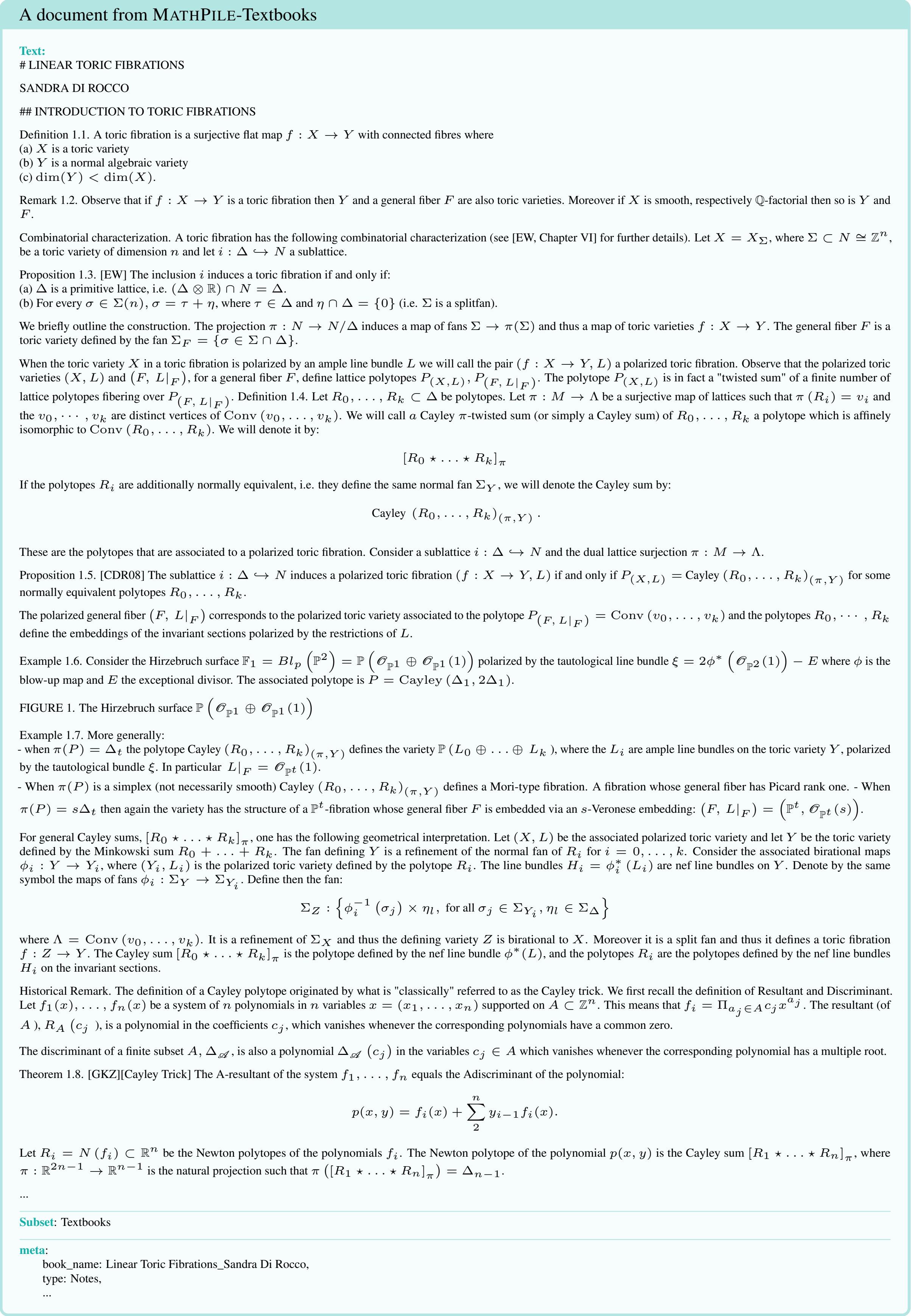
symbols_to_words_ratio: 0.0,
uppercase_word_ratio: 0.0871,
...

Figure 7: An example Wikipedia document in MathPile



A document from MATHPILE-Textbooks

Text:
# LINEAR TORIC FIBRATIONS

SANDRA DI ROCCO

## INTRODUCTION TO TORIC FIBRATIONS

Definition 1.1. A toric fibration is a surjective flat map $f : X \rightarrow Y$ with connected fibres where
(a) X is a toric variety
(b) Y is a normal algebraic variety
(c) $\dim(Y) < \dim(X)$.

Remark 1.2. Observe that if $f : X \rightarrow Y$ is a toric fibration then Y and a general fiber F are also toric varieties. Moreover if X is smooth, respectively $\mathbb{Q}$-factorial then so is Y and F.

Combinatorial characterization. A toric fibration has the following combinatorial characterization (see [EW, Chapter VI] for further details). Let $X = X_{\Sigma}$, where $\Sigma \subset N \cong \mathbb{Z}^{n}$, be a toric variety of dimension n and let $i : \Delta \hookrightarrow N$ a sublattice.

Proposition 1.3. [EW] The inclusion i induces a toric fibration if and only if:
(a) Δ is a primitive lattice, i.e. $(\Delta \otimes \mathbb{R}) \cap N = \Delta$.
(b) For every $\sigma \in \Sigma(n)$, $\sigma = \tau + \eta$, where $\tau \in \Delta$ and $\eta \cap \Delta = \{0\}$ (i.e. Σ is a splitfan).

We briefly outline the construction. The projection $\pi : N \rightarrow N / \Delta$ induces a map of fans $\Sigma \rightarrow \pi(\Sigma)$ and thus a map of toric varieties $f : X \rightarrow Y$. The general fiber F is a toric variety defined by the fan $\Sigma_{F} = \{\sigma \in \Sigma \cap \Delta\}$.

When the toric variety X in a toric fibration is polarized by an ample line bundle L we will call the pair $(f : X \rightarrow Y, L)$ a polarized toric fibration. Observe that the polarized toric varieties (X, L) and $\left(F, L|_{F}\right)$, for a general fiber F, define lattice polytopes $P_{(X, L)}, P_{\left(F, L|_{F}\right)}$. The polytope $P_{(X, L)}$ is in fact a "twisted sum" of a finite number of lattice polytopes fibering over $P_{\left(F, L|_{F}\right)}$. Definition 1.4. Let $R_{0}, \ldots, R_{k} \subset \Delta$ be polytopes. Let $\pi : M \rightarrow \Lambda$ be a surjective map of lattices such that $\pi\left(R_{i}\right) = v_{i}$ and the $v_{0}, \cdots, v_{k}$ are distinct vertices of $\operatorname{Conv}\left(v_{0}, \ldots, v_{k}\right)$. We will call a Cayley π-twisted sum (or simply a Cayley sum) of $R_{0}, \ldots, R_{k}$ a polytope which is affinely isomorphic to $\operatorname{Conv}\left(R_{0}, \ldots, R_{k}\right)$. We will denote it by:

$$\left[R_{0} \star \ldots \star R_{k}\right]_{\pi}$$

If the polytopes R_{i} are additionally normally equivalent, i.e. they define the same normal fan Σ_{Y}, we will denote the Cayley sum by:

$$\text{Cayley}\ \left(R_{0}, \ldots, R_{k}\right)_{(\pi, Y)} .$$

These are the polytopes that are associated to a polarized toric fibration. Consider a sublattice $i : \Delta \hookrightarrow N$ and the dual lattice surjection $\pi : M \rightarrow \Lambda$.

Proposition 1.5. [CDR08] The sublattice $i : \Delta \hookrightarrow N$ induces a polarized toric fibration $(f : X \rightarrow Y, L)$ if and only if $P_{(X, L)} =$ Cayley $\left(R_{0}, \ldots, R_{k}\right)_{(\pi, Y)}$ for some normally equivalent polytopes $R_{0}, \ldots, R_{k}$.

The polarized general fiber $\left(F, L|_{F}\right)$ corresponds to the polarized toric variety associated to the polytope $P_{\left(F, L|_{F}\right)} = \operatorname{Conv}\left(v_{0}, \ldots, v_{k}\right)$ and the polytopes $R_{0}, \cdots, R_{k}$ define the embeddings of the invariant sections polarized by the restrictions of L.

Example 1.6. Consider the Hirzebruch surface $\mathbb{F}_{1} = B l_{p}\left(\mathbb{P}^{2}\right) = \mathbb{P}\left(\mathscr{O}_{\mathbb{P}^{1}} \oplus \mathscr{O}_{\mathbb{P}^{1}}(1)\right)$ polarized by the tautological line bundle $\xi = 2 \phi^{*}\left(\mathscr{O}_{\mathbb{P}^{2}}(1)\right) - E$ where ϕ is the blow-up map and E the exceptional divisor. The associated polytope is $P = \operatorname{Cayley}\left(\Delta_{1}, 2 \Delta_{1}\right)$.

FIGURE 1. The Hirzebruch surface $\mathbb{P}\left(\mathscr{O}_{\mathbb{P}^{1}} \oplus \mathscr{O}_{\mathbb{P}^{1}}(1)\right)$

Example 1.7. More generally:
- when $\pi(P) = \Delta_{t}$ the polytope Cayley $\left(R_{0}, \ldots, R_{k}\right)_{(\pi, Y)}$ defines the variety $\mathbb{P}\left(L_{0} \oplus \ldots \oplus L_{k}\right)$, where the L_{i} are ample line bundles on the toric variety Y, polarized by the tautological bundle ξ. In particular $\left.L\right|_{F} = \mathscr{O}_{\mathbb{P}^{t}}(1)$.
- When $\pi(P)$ is a simplex (not necessarily smooth) Cayley $\left(R_{0}, \ldots, R_{k}\right)_{(\pi, Y)}$ defines a Mori-type fibration. A fibration whose general fiber has Picard rank one. - When $\pi(P) = s \Delta_{t}$ then again the variety has the structure of a $\mathbb{P}^{t}$-fibration whose general fiber F is embedded via an s-Veronese embedding: $\left(F, L|_{F}\right) = \left(\mathbb{P}^{t}, \mathscr{O}_{\mathbb{P}^{t}}(s)\right)$.

For general Cayley sums, $\left[R_{0} \star \ldots \star R_{k}\right]_{\pi}$, one has the following geometrical interpretation. Let (X, L) be the associated polarized toric variety and let Y be the toric variety defined by the Minkowski sum $R_{0} + \ldots + R_{k}$. The fan defining Y is a refinement of the normal fan of R_{i} for $i = 0, \ldots, k$. Consider the associated birational maps $\phi_{i} : Y \rightarrow Y_{i}$, where $\left(Y_{i}, L_{i}\right)$ is the polarized toric variety defined by the polytope R_{i}. The line bundles $H_{i} = \phi_{i}^{*}\left(L_{i}\right)$ are nef line bundles on Y. Denote by the same symbol the maps of fans $\phi_{i} : \Sigma_{Y} \rightarrow \Sigma_{Y_{i}}$. Define then the fan:

$$\Sigma_{Z} : \left\{\phi_{i}^{-1}\left(\sigma_{j}\right) \times \eta_{l}, \text{ for all } \sigma_{j} \in \Sigma_{Y_{i}}, \eta_{l} \in \Sigma_{\Delta}\right\}$$

where $\Lambda = \operatorname{Conv}\left(v_{0}, \ldots, v_{k}\right)$. It is a refinement of Σ_{X} and thus the defining variety Z is birational to X. Moreover it is a split fan and thus it defines a toric fibration $f : Z \rightarrow Y$. The Cayley sum $\left[R_{0} \star \ldots \star R_{k}\right]_{\pi}$ is the polytope defined by the nef line bundle $\phi^{*}(L)$, and the polytopes R_{i} are the polytopes defined by the nef line bundles H_{i} on the invariant sections.

Historical Remark. The definition of a Cayley polytope originated by what is "classically" referred to as the Cayley trick. We first recall the definition of Resultant and Discriminant. Let $f_{1}(x), \ldots, f_{n}(x)$ be a system of n polynomials in n variables $x = \left(x_{1}, \ldots, x_{n}\right)$ supported on $A \subset \mathbb{Z}^{n}$. This means that $f_{i} = \Pi_{a_{j} \in A} c_{j} x^{a_{j}}$. The resultant (of A), $R_{A}\left(c_{j}\right.$), is a polynomial in the coefficients c_{j}, which vanishes whenever the corresponding polynomials have a common zero.

The discriminant of a finite subset A, $\Delta_{\mathscr{A}}$, is also a polynomial $\Delta_{\mathscr{A}}\left(c_{j}\right)$ in the variables $c_{j} \in A$ which vanishes whenever the corresponding polynomial has a multiple root.

Theorem 1.8. [GKZ][Cayley Trick] The A-resultant of the system $f_{1}, \ldots, f_{n}$ equals the Adiscriminant of the polynomial:

$$p(x, y) = f_{i}(x) + \sum_{2}^{n} y_{i-1} f_{i}(x) .$$

Let $R_{i} = N\left(f_{i}\right) \subset \mathbb{R}^{n}$ be the Newton polytopes of the polynomials f_{i}. The Newton polytope of the polynomial $p(x, y)$ is the Cayley sum $\left[R_{1} \star \ldots \star R_{n}\right]_{\pi}$, where $\pi : \mathbb{R}^{2 n-1} \rightarrow \mathbb{R}^{n-1}$ is the natural projection such that $\pi\left(\left[R_{1} \star \ldots \star R_{n}\right]_{\pi}\right) = \Delta_{n-1}$.

...

Subset: Textbooks

meta:
book_name: Linear Toric Fibrations_Sandra Di Rocco,
type: Notes,
...

Figure 8: An example textbook document in MATHPILE

A document from MATHPILE-ProofWiki

Text:
\section{Test for Submonoid}
Tags: Abstract Algebra, Monoids

\begin{theorem}
To show that \struct {T, circ} is a submonoid of a monoid \struct {S, circ}, we need to show that:
:(1): $T \subseteq S$
:(2): \struct {T, circ} is a magma (that is, that it is closed)
:(3): \struct {T, circ} has an identity.
\end{theorem}

\begin{proof}
From Subsemigroup Closure Test, (1) and (2) are sufficient to show that \struct {T, circ} is a subsemigroup of \struct {S, circ}.
Demonstrating the presence of an identity is then sufficient to show that it is a monoid. {{qed}}
Category:Monoids
\end{proof}

...

Subset: ProofWiki

meta:
type: Theorem_Proof,
...

Figure 9: An example ProofWiki (a theorem and its proof) document in MATHPILE

A document from MATHPILE-ProofWiki

Text:
\begin{definition}[Definition:That which produces Medial Whole with Medial Area/Whole]

Let $a, b \in \mathcal{R}_{>0}$ be (strictly) positive real numbers such that $a > b$.

Let $a - b$ be a straight line which produces with a medial area a medial whole.

The real number a is called the "'whole"' of the straight line which produces with a medial area a medial whole.

Category:Definitions/Euclidean Number Theory

\end{definition}

Subset: ProofWiki

meta:
type: Definition,
...

Figure 10: An example ProofWiki (definition) document in MATHPILE

A document from MathPile-arXiv

Text:

\begin{document}
\title{Coherence freeze in an optical lattice investigated via pump-probe spectroscopy}
\author{Samansa Maneshi}
\email[]{smaneshi@physics.utoronto.ca}
\author{Chao Zhuang}
\author{Christopher R. Paul}
\author{Luciano S. Cruz}
\altaffiliation[Current address:]{UFABC, São Paulo, Brazil.}
\author{Aephraim M. Steinberg}
\affiliation{Centre for Quantum Information & Quantum Control and Institute for Optical Sciences,
Department of Physics, University of Toronto, Canada }
\date{\today}
\pacs{37.10.Jk, 03.65.Yz, 03.67.-a, 42.50.Md}

\begin{abstract}
Motivated by our observation of fast echo decay and a surprising coherence freeze, we have developed a pump-probe spectroscopy technique for vibrational states of ultracold ^{85}Rb atoms in an optical lattice to gain information about the memory dynamics of the system. We use pump-probe spectroscopy to monitor the time-dependent changes of frequencies experienced by atoms and to characterize the probability distribution of these frequency trajectories. We show that the inferred distribution, unlike a naive microscopic model of the lattice, correctly predicts the main features of the observed echo decay.
\end{abstract}

\maketitle

Characterizing decoherence mechanisms is a crucial task for experiments aiming to control quantum systems, e.g., for quantum information processing (QIP). In this work, we demonstrate how two-dimensional (2D) pump-probe spectroscopy may be extended to provide important information on these mechanisms. As a model system, we study quantum vibrational states of ultracold atoms in an optical lattice. In addition to being a leading candidate system for QIP \citeBrennenJaksch, optical lattices are proving a versatile testing ground for the development of quantum measurement and control techniques \citeOMandel, Anderlini and a powerful tool for quantum simulations, e.g. the study of Anderson localization and the Hubbard model \citeMottAnderson.

In our experiment, we study the vibrational coherence of ^{85}Rb atoms trapped in a shallow one-dimensional standing wave. Through our 2D pump-probe technique, we obtain detailed microscopic information on the frequency drift experienced by atoms in the lattice, enabling us to predict the evolution of coherence. Since the pioneering development of the technique in NMR\citeJeener-Ernst, 2D spectroscopy has been widely used to obtain high-resolution spectra and gain information about relaxations, couplings, and many-body interactions, in realms ranging from NMR \citeErnst to molecular spectroscopy \citeMukamel-Jonas, Hybl, Brixner, MillerNature to semiconductor quantum wells \citeCundiff, KWStone. Here, we show that similar powerful techniques can be applied to the quantized center-of-mass motion of trapped atoms, and more generally, offer a new tool for the characterization of systems in QIP and quantum control.

\begin{figure}
\caption{(Color online) Two typical measurements of echo amplitude vs. time. The echo pulse and the observed echo envelope are centered at times t_p and $2t_p$, respectively. After an initial decay, echo amplitude stays constant for about $1ms$ forming a plateau, before decaying to zero. The average lattice depths are $20E_R$ (circles) and $18E_R$ (squares).}
\label{fig1}
\end{figure}

We have previously measured the evolution of coherence between the lowest two vibrational states of potential wells \cite{Ours}.

The dephasing time is about $0.3ms$ ($T_2^\star$).

This dephasing is partly due to an inhomogeneous distribution of lattice depths as a result of the transverse Gaussian profile of the laser beams. To measure the homogeneous decoherence time (T_2), we perform pulse echoes, measuring the echo amplitude as a function of time \cite{Ours}.

Figure \ref{fig1} shows two typical measurements of echo amplitude carried out on different dates under slightly different conditions such as different average lattice depths and different dephasing times. The echo amplitude initially decays with a time constant of about $0.7ms$, which is much faster than the photon scattering time ($\sim 60ms$) in the lattice. It then exhibits a $1ms$-long coherence freeze followed by a final decay. Absent real decoherence on the short time scale of $1ms$, only loss of frequency memory would inhibit the appearance of echoes. This loss comes about when atoms experience time-varying frequencies. We use 2D pump-probe spectroscopy to monitor this frequency drift. Our 2D pump-probe spectroscopy is essentially a version of spectral hole-burning for vibrational states. By monitoring the changes in the hole spectrum as a function of time we gain information on the atoms' frequency drift.

Information obtained from our 2D spectra enables us to characterize the temporal decay of frequency memory and through our simulations we find that “coherence freeze" is related to the shape of this memory loss function.

Similar plateaus in echo decay and a two-stage decay of echo amplitude have been observed in a Cooper-pair box \cite{Nakamura}, for a single electron spin in a quantum dot \cite{Vandersypen} and for electron spins in a semiconductor \cite{SClark}. Those plateaus or two-stage decays have been either explained through {\it{a priori}} models or simply described phenomenologically. Here, we are introducing an experimental technique to directly probe the origin of plateaus.

The periodic potential in our experiment is formed by interfering two laser beams blue-detuned by $25GHz$ from the D2 transition line, $F=3\to F'=4$ ($\lambda=780nm$), thus trapping atoms in the regions of low intensity, which minimizes the photon scattering rate and the transverse forces. The two laser beams intersect with parallel linear polarizations at an angle of $\theta=(49.0\pm0.2)^\circ$, resulting in a spacing of $L=(0.930\pm0.004)\mu m$ between the wells. Due to gravity, the full effective potential also possesses a “tilt” of $2.86E_R$ per lattice site, where $E_R=\frac{h^2}{8mL^2}$ is the effective lattice recoil energy. The photon scattering time in our experiment is $\approx 60ms$ and the Landau-Zenner tunneling times for transitions from the lowest two levels are greater than $160ms$.
Atoms are loaded to the lattice during a molasses cooling stage and prepared in the ground vibrational state by adiabatic filtering \cite{StefanQPT}. Due to the short coherence length of atoms in optical molasses ($60nm$ at $10\mu K$), there is no coherence between the wells. We measure populations of atoms in the ground vibrational, the first excited, and the (lossy) higher excited states P_1, P_2, and P_L, respectively, by fluorescence imaging of the atomic cloud after adiabatic filtering \cite{StefanQPT}.

...

Subset: arXiv

meta:
id: 1005.2635,
language_detection_score: 0.8389,
...

Figure 11: An example arXiv document in MathPile

A document from MATHPILE-StackExchange

Question:
Title: Are fractions hard because they are like algebra?
Body:
It occurs to me that to really understand the ways that people work with fractions on paper requires a good grasp of the ideas that numbers have multiple representations and that expressions can be manipulated in various ways without changing the number they represent. These are essentially algebraic ideas.
For example, adding fractions requires us to rewrite the fractions in a different form, and then essentially factorise the expression. This is the same as rearranging expressions in algebra. Dividing fractions requires us to rerepresent an operation like $\div \frac{2}{3}$ as $\times \frac{3}{2}$. This is the same as realising the connection between operations that you use to solve equations in algebra. And cancelling down before multiplying is very sophisticated rewriting relying on various associative and commutative laws.
So it seems that we are really asking children to think in algebraic ways in order to understand fraction calculations well. This would seem to me to be a good reason why children and adults find it hard - they need more scaffolding in some abstract ideas.
Is this a reasonable theory and has anyone written about this algebra-fractions connection before? To be clear, I am not asking if this is the only reason fractions are hard, but if there is any discussion out there to draw parallels between learning algebra and learning to manipulate fractions.
Id: 7826
Score: 17
Tags: <algebra><fractions>
LastEditorDisplayName: None
OwnerDisplayName: None
ClosedDate: None
FavoriteCount: None
language_detection_score: 0.9558

...

Answers:

Body: Not sure about paper references. One reason why people dont́ understand fractions is because they are seemingly illogical.

You score one basket out of three 1/3.

A little while later you try again and score 1/2. Clearly you have scored 2/5 shots? In many ways this is the correct answer. So why shouldnt́ $\frac{1}{3} + \frac{1}{2} = \frac{2}{5}$

People generally dont́ understand equivalent fractions. It is strange for one farmer to say there are 4 sheep and another to say there are 8/2 sheep in the same field. People assume that the number 4 does what it says on the tin and is how we always describe 4 ness of something. They dont́ understand equivalence.

Partly to blame is treating fractions like conjuring tricks. If this is the question...do this, if this is the question ...do another uncorrelated thing. I asked my class (who seemingly could compute $\frac{2}{3} \times \frac{3}{5}$ correctly) to draw me a picture

instead of just multiplying. No one could do it yet they all said "but itś
$frac615$ you times the top and the bottom!"

I think drawing fractions is extremely useful. Draw $\frac{2}{3} \div 2$ or $2 \div \frac{2}{3}$ Itś not easy but I find students develop robustness eventually and begin to abstract themselves.

Id: 7827,
Score: 9,
is_accepted_answer: False,
language_detection_score: 0.9599,

Body: The obvious (to me) source of difficulty is that fractions are just plain complicated, more so than almost anything else in elementary education. You have to operate with a pair of numbers, instead of a single one, and you have to keep the order straight. Adding is quite complicated in its own right. Things are further complicated by rules about least common denominators and least terms.

Iḿ a little unclear about the questionś emphasis on algebra. Any sort of general rule or operation in arithmetic must have a connection to algebra, but I do not see what is intrinsically difficult about algebra that relates to numeric fractions. Certainly some parts of algebra are hard, and some parts harder than others, algebraic fractions among them. It seems to me that fractions are difficult because itś easy to confuse the various bits. Even when youv́e got them straight, theyŕe noticeably slower to use, take concentration, and when things have such cognitive demands, theyŕe harder to think with.

Conceptually, theyŕe a little bit odd, which is probably distracting until you get used to them. What they represent do not seem to apply to the same things that (whole) numbers do. Evidently fractions are not considered in this passage:

In that city, which was the oldest in the world, the cat was an object of veneration. Its worship was the religion of the country. The multiplication and addition of cats were a perpetual instruction in arithmetic. Naturally, any inattention to the wants of a cat was punished with great severity in this world and the next... – A. Bierce, "A Revolt of the Gods"

Now to have one-and-a-half cats seems a very different thing than to have three halves. In the former case, thereś a good chance that the one cat you have will be alive and purring, while the same could not possibly be said about any of the halves. No doubt such lessons are considered blasphemous in that city. While many things may be divided into parts – cars are a better example than cats – not many can be divided into equivalent parts that can be used as a basis for fractions. As we get used to fractions, as well as real numbers, we are taught to ignore this and accept statements such as "the average family has 2.4 children." Here is another example:

By then, she will have shed 80 of the 240 pounds she weighed in with when she entered Peter Bent Brigham hospital obesity program. A third of her left behind! – The Boston Herald American, 7/7/77

The question seems to welcome references. There are certainly several that connect fractions with algebra. This paper,

Seigler et al. (2013), Fractions: the new frontier for theories of numerical development, Trends in Cognitive Sciences,

is a short survey of what is known and unknown about neural bases for oneś knowledge of fractions. Whole number arithmetic knowledge has been studied, and the authors suggest that the representation of the knowledge fractions is an area ripe for investigation. It reviews (with references) why fractions are difficult and the relation of skill at fractions to skill at algebra. Generally – or, rather, I only know of papers that discuss the connection in that direction, with algebra skill being dependent on fractions skill. (OTOH, Iḿ not widely read in this area.)

Id: 7831,
Score: 11,
is_accepted_answer: False,
language_detection_score: 0.9780

Subset: StackExchange

Figure 12: An example StackExchange document in MATHPILE. Here is a question from "matheducators" ".stackexchang.com" with two high-quality responses.

In algebraic topology we often encounter chain complexes with extra multiplicative structure. For example, the cochain complex of a topological space has what is called the E_∞-algebra structure which comes from the cup product. *In this talk I present an idea for studying such chain complexes, E_∞ differential graded algebras (E_∞ DGAs), using stable homotopy theory. Namely, I discuss new equivalences between E_∞ DGAS that are defined using commutative ring spectra.* ring spectra are equivalent. *Quasi-isomorphic E_∞ DGAs are E_∞ topologically equivalent. However, the examples I am going to present show that the opposite is not true; there are E_∞ DGAs that are E_∞ topologically equivalent but not quasi-isomorphic. This says that between E_∞ DGAs, we have more equivalences than just the quasi-isomorphisms. I also discuss interaction of E_∞ topological equivalences with the Dyer-Lashof operations and cases where E_∞topological equivalences and quasi-isomorphisms agree.*	Özet : *In algebraic topology we often encounter chain complexes with extra multiplicative structure. For example, the cochain complex of a topological space has what is called the E_∞-algebra structure which comes from the cup product. In this talk I present an idea for studying such chain complexes, E_∞ differential graded algebras (E_∞ DGAs), using stable homotopy theory. Namely, I discuss new equivalences between E_∞ DGAS that are defined using commutative ring spectra*.We say E_∞ DGAs are E_∞ topologically equivalent when the corresponding commutative ring spectra are equivalent. *Quasi-isomorphic E_∞ DGAs are E_∞ topologically equivalent. However, the examples I am going to present show that the opposite is not true; there are E_∞ DGAs that are E_∞ topologically equivalent but not quasi-isomorphic. This says that between E_∞ DGAs, we have more equivalences than just the quasi-isomorphisms. I also discuss interaction of E_∞ topological equivalences with the Dyer-Lashof operations and cases where E_∞ topological equivalences and quasi-isomorphisms agree.*
Université de la Saskatchewan, 1 - 4 juin 2015 www.smc.math.ca//2015f Comité d'organisation Financement étudiants Minisymposia invités Minisymposia libres Conférences libres Horaire - Minisymposa invités Open Problems Graphs and matrices Responsable et président: Shaun Fallat et Karen Meagher (University of Regina) *WAYNE BARRETT, Brigham Young University* *The Fielder Vector and Tree Decompositions of Graphs [PDF]* *In the 1970's Fiedler initiated a study of the second smallest eigenvalue of the Laplacian matrix L of a graph and the corresponding eigenvector(s). These "Fiedler" vectors have become spectacularly successful in revealing properties of the associated graph. A tree decomposition $\mathcal{T}$ of a graph $G = (V, E)$ is an associated tree whose nodes are subsets of V and whose edge set respects the structure of G. Tree decompositions have been used in the analysis of complex networks. This talk reports on an algorithm developed by students at BYU for obtaining a tree decomposition by means of Fiedler vector(s) of G.* *...* *Graphs that have a weighted adjacency matrix with spectrum $\{\lambda_1^{n-2}, \lambda_2^2\}$ [PDF]* *In this talk I will characterize the graphs which have an edge weighted adjacency matrix belonging to the class of $n \times n$ involutions with spectrum equal to $\{\lambda_1^{n-2}, \lambda_2^2\}$ for some λ_1 and some λ_2. The connected graphs turn out to be the cographs constructed as the join of at least two unions of pairs of complete graphs, and possibly joined with one other complete graph.*	University of Saskatchewan, June 1 - 4, 2015 www.cms.math.ca//2015 Invited Minisymposia Contributed Minisymposia Contributed Talks Graphs and matrices Organizer and Chair: Shaun Fallat and Karen Meagher (University of Regina) *WAYNE BARRETT, Brigham Young University* *The Fielder Vector and Tree Decompositions of Graphs [PDF]* *In the 1970's Fiedler initiated a study of the second smallest eigenvalue of the Laplacian matrix L of a graph and the corresponding eigenvector(s). These "Fiedler" vectors have become spectacularly successful in revealing properties of the associated graph. A tree decomposition $\mathcal{T}$ of a graph $G = (V, E)$ is an associated tree whose nodes are subsets of V and whose edge set respects the structure of G. Tree decompositions have been used in the analysis of complex networks. This talk reports on an algorithm developed by students at BYU for obtaining a tree decomposition by means of Fiedler vector(s) of G.* *...* *Graphs that have a weighted adjacency matrix with spectrum $\{\lambda_1^{n-2}, \lambda_2^2\}$ [PDF]* *In this talk I will characterize the graphs which have an edge weighted adjacency matrix belonging to the class of $n \times n$ involutions with spectrum equal to $\{\lambda_1^{n-2}, \lambda_2^2\}$ for some λ_1 and some λ_2. The connected graphs turn out to be the cographs constructed as the join of at least two unions of pairs of complete graphs, and possibly joined with one other complete graph.*

Table 8: Near-duplication matches found in CommonCrawl by MinHash LSH deduplication (in *italics*).

<table>
<tr>
<td>\begin{document}

\title{Querying Guarded Fragments via Resolution}

\section{A detailed example}

Here we include some equations and theorem-like environments to show
how these are labeled in a supplement and can be referenced from the
main text.
Consider the following equation:
\begin{equation}
\label{eq:suppa}
a^2 + b^2 = c^2.
\end{equation}
You can also reference equations such as \cref{eq:matrices,eq:bb}
from the main article in this supplement.
\lipsum[100-101]

\begin{theorem}
An example theorem.
\end{theorem}

\lipsum[102]

\begin{lemma}
An example lemma.
\end{lemma}

\lipsum[103-105]

Here is an example citation: \cite{KoMa14}.

\section[Proof of Thm][Proof of \cref{thm:bigthm}}
\label{sec:proof}

\lipsum[106-112]

\section{Additional experimental results}
\Cref{tab:foo} shows additional
supporting evidence.

\begin{table}[htbp]
{\footnotesize
\caption{Example table} \label{tab:foo}
\begin{center}
\begin{tabular}{|c|c|c|} \hline
 Species & \bf Mean & \bf Std.~Dev. \\\hline
 1 & 3.4 & 1.2 \\
 2 & 5.4 & 0.6 \\\hline
\end{tabular}
\end{center}
}
\end{table}

\end{document}</td>
<td>\begin{document}

\title{Limited memory Kelley’s Method Converges for Composite Convex and Submodular Objectives}

\section{A detailed example}

Here we include some equations and theorem-like environments to show
how these are labeled in a supplement and can be referenced from the
main text.
Consider the following equation:
\begin{equation}
\label{eq:suppa}
a^2 + b^2 = c^2.
\end{equation}
You can also reference equations such as \cref{eq:matrices,eq:bb}
from the main article in this supplement.
\lipsum[100-101]

\begin{theorem}
An example theorem.
\end{theorem}

\lipsum[102]

\begin{lemma}
An example lemma.
\end{lemma}

\lipsum[103-105]

Here is an example citation: \cite{KoMa14}.

\section[Proof of Thm][Proof of \cref{thm:bigthm}}
\label{sec:proof}

\lipsum[106-112]

\section{Additional experimental results}
\Cref{tab:foo} shows additional
supporting evidence.

\begin{table}[htbp]
{\footnotesize
\caption{Example table} \label{tab:foo}
\begin{center}
\begin{tabular}{|c|c|c|} \hline
 Species & \bf Mean & \bf Std.~Dev. \\\hline
 1 & 3.4 & 1.2 \\
 2 & 5.4 & 0.6 \\\hline
\end{tabular}
\end{center}
}
\end{table}

\end{document}</td>
</tr>
</table>

Table 9: A near-duplication match found in arXiv by MinHashLSH deduplication (in *italics*).

```
\section{Definition:Constructed Semantics
    /Instance 4/Rule of Idempotence}
Tags: Formal Semantics

\begin{theorem}
The Rule of Idempotence:
:$(p \lor p) \implies p$
is a tautology in Instance 4 of
    constructed semantics.
\end{theorem}

\begin{proof}
By the definitional abbreviation for the
    conditional:
:$\mathbf A \implies \mathbf B =_{\text{
    def}} \neg \mathbf A \lor \mathbf B$
the Rule of Idempotence can be written as
    :
: $\neg \left({p \lor p}\right) \lor p$
This evaluates as follows:
:$\begin{array}{|cccc|c|c|} \hline
\neg & (p & \lor & p) & \lor & p \\
\hline
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 \\
0 & 2 & 2 & 2 & 0 & 2 \\
2 & 3 & 3 & 3 & 0 & 3 \\
\hline
\end{array}$
{{qed}}
Category:Formal Semantics
\end{proof}
```

```
\section{Definition:Constructed Semantics
    /Instance 5/Rule of Idempotence}
Tags: Formal Semantics

\begin{theorem}
The Rule of Idempotence:
:$(p \lor p) \implies p$
is a tautology in Instance 5 of
    constructed semantics.
\end{theorem}

\begin{proof}
By the definitional abbreviation for the
    conditional:
:$\mathbf A \implies \mathbf B =_{\text{
    def}} \neg \mathbf A \lor \mathbf B$
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:$\begin{array}{|cccc|c|c|} \hline
\neg & (p & \lor & p) & \lor & p \\
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1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 \\
3 & 2 & 2 & 2 & 0 & 2 \\
0 & 3 & 3 & 3 & 0 & 3 \\
\hline
\end{array}$
{{qed}}
Category:Formal Semantics
\end{proof}
```

```
\section{Imaginary Part of Complex
    Product}
Tags: Complex Multiplication

\begin{theorem}
Let $z_1$ and $z_2$ be complex numbers.
Then:
:$\map \Im {z_1 z_2} = \map \Re {z_1} \,
    \map \Im {z_2} + \map \Im {z_1} \, \
    map \Re {z_2}$
\end{theorem}

\begin{proof}
Let $z_1 = x_1 + i y_1$ and $z_2 = x_2 +
    i y_2$.
By definition of complex multiplication:
:$z_1 z_2 = x_1 x_2 - y_1 y_2 + i \paren
    {x_1 y_2 + x_2 y_1}$
Then
{{begin-eqn}}
{{eqn | l = \map \Im {z_1 z_2}
      | r = x_1 y_2 + x_2 y_1
      | c = {{Defof|Imaginary Part}}
}}
{{eqn | r = \map \Re {z_1} \, \map \Im {
    z_2} + \map \Im {z_1} \, \map \Re {z_2
    }
      | c = {{Defof|Imaginary Part}}
}}
{{end-eqn}}
{{qed}}
\end{proof}
```

```
\section{Real Part of Complex Product}
Tags: Complex Multiplication

\begin{theorem}
Let $z_1$ and $z_2$ be complex numbers.
Then:
:$\map \Re {z_1 z_2} = \map \Re {z_1} \
    map \Re {z_2} - \map \Im {z_1} \map \
    Im {z_2}$
\end{theorem}

\begin{proof}
Let $z_1 = x_1 + i y_1$ and $z_2 = x_2 +
    i y_2$.
By definition of complex multiplication:
:$z_1 z_2 = x_1 x_2 - y_1 y_2 + i \paren
    {x_1 y_2 + x_2 y_1}$
Then:
{{begin-eqn}}
{{eqn | l = \map \Re {z_1 z_2}
      | r = x_1 x_2 - y_1 y_2
      | c = {{Defof|Real Part}}
}}
{{eqn | r = \map \Re {z_1} \map \Re {z_2}
     - \map \Im {z_1} \map \Im {z_2}
      | c = {{Defof|Real Part}}
}}
{{end-eqn}}
{{qed}}
\end{proof}
```

Table 10: Near-duplication matches found in ProofWiki by MinHash LSH deduplication.

<table>
<tr>
<td># HP-42S

The **HP-42S RPN Scientific** is a programmable RPN Scientific hand held calculator introduced by Hewlett-Packard in 1988. It has advanced functions suitable for applications in mathematics, linear algebra, statistical analysis, computer science and others.

HP-42S
The HP-42S
—
Type| Programmable scientific
Manufacturer| Hewlett-Packard
Introduced| 1988
Discontinued| 1995 Calculator
Entry model| RPN
Precision| 12 display digits (15 digits internally),[1] exponent ±499
Display type| LCD dot-matrix
Display size| 2 lines, 22 characters, 131×16 pixels CPU
Processor| Saturn (Lewis) Programming
Programming language(s)| RPN key stroke (fully merged)
Firmware memory| 64 KB of ROM
Program steps| 7200 Interfaces
Ports| IR (Infrared) printing Other
Power supply| 3×1.5V button cell batteries (Panasonic LR44, Duracell PX76A/675A or Energizer 357/303)
Weight| 6 oz (170 g)
Dimensions| 148×80×15mm

Overview

Perhaps the HP-42S was to be released as a replacement for the aging HP-41 series as it is designed to be compatible with all programs written for the HP-41. Since it lacked expandability, and lacked any real I/O ability, both key features of the HP-41 series, it was marketed as an HP-15C replacement.

The 42S, however, has a much smaller form factor than the 41, and features many more built-in functions, such as a matrix editor, complex number support, an equation solver, user-defined menus, and basic graphing capabilities (the 42S can draw graphs only by programs). Additionally, it features a two-line dot matrix display, which made stack manipulation easier to understand.

Production of the 42S ended in 1995.[2] As this calculator is regarded amongst the best ever made in terms of quality, key stroke feel, ease of programming, and daily usability for engineers,[3] in the HP calculator community the 42S has become famous for its high prices in online auctions, up to several times its introduction price, which has created a scarcity for utility end users.</td>
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HP-42S
The HP-42S
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Type| Programmable scientific
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Production of the 42S ended in 1995.[2] As this calculator is regarded amongst the best ever made in terms of quality, key stroke feel, ease of programming, and daily usability for engineers,[3] in the HP calculator community the 42S has become famous for its high prices in online auctions, up to several times its introduction price, which has created a scarcity for utility end users.</td>
</tr>
</table>

Table 11: Duplication matches found in Wikipedia by MinHash LSH deduplication (in *italics*).

# Basic Concepts in Graph Theory	*# Basic Concepts in Graph Theory*
## Section 1: What is a Graph?	*## Section 1: What is a Graph?*
There are various types of graphs, each with its own definition. Unfortunately, some people apply the term "graph" rather loosely, so you can't be sure what type of graph they're talking about unless you ask them. After you have finished this chapter, we expect you to use the terminology carefully, not loosely. To motivate the various definitions, we'll begin with some examples.	*There are various types of graphs, each with its own definition. Unfortunately, some people apply the term "graph" rather loosely, so you can't be sure what type of graph they're talking about unless you ask them. After you have finished this chapter, we expect you to use the terminology carefully, not loosely. To motivate the various definitions, we'll begin with some examples.*
Example 1 (A computer network) Computers are often linked with one another so that they can interchange information. Given a collection of computers, we would like to describe this linkage in fairly clean terms so that we can answer questions such as "How can we send a message from computer A to computer B using the fewest possible intermediate computers?"	*Example 1 (A computer network) Computers are often linked with one another so that they can interchange information. Given a collection of computers, we would like to describe this linkage in fairly clean terms so that we can answer questions such as "How can we send a message from computer A to computer B using the fewest possible intermediate computers?"*
We could do this by making a list that consists of pairs of computers that are connected. Note that these pairs are unordered since, if computer C *can communicate with computer* D*, then the reverse is also true. (There are sometimes exceptions to this, but they are rare and we will assume that our collection of computers does not have such an exception.) Also, note that we have implicitly assumed that the computers are distinguished from each other: It is insufficient to say that "A PC is connected to a Mac." We must specify which* PC *and which Mac. Thus, each computer has a unique identifying label of some sort.*	*We could do this by making a list that consists of pairs of computers that are connected. Note that these pairs are unordered since, if computer* C *can communicate with computer* D*, then the reverse is also true. (There are sometimes exceptions to this, but they are rare and we will assume that our collection of computers does not have such an exception.) Also, note that we have implicitly assumed that the computers are distinguished from each other: It is insufficient to say that "A PC is connected to a Mac." We must specify which* PC *and which Mac. Thus, each computer has a unique identifying label of some sort.*
For people who like pictures rather than lists, we can put dots on a piece of paper, one for each computer. We label each dot with a computer's identifying label and draw a curve connecting two dots if and only if the corresponding computers are connected. Note that the shape of the curve does not matter (it could be a straight line or something more complicated) because we are only interested in whether two computers are connected or not. Below are two such pictures of the same graph. Each computer has been labeled by the initials of its owner.	*For people who like pictures rather than lists, we can put dots on a piece of paper, one for each computer. We label each dot with a computer's identifying label and draw a curve connecting two dots if and only if the corresponding computers are connected. Note that the shape of the curve does not matter (it could be a straight line or something more complicated) because we are only interested in whether two computers are connected or not. Below are two such pictures of the same graph. Each computer has been labeled by the initials of its owner.*
...	*...*
## Basic Concepts in Graph Theory	*## Basic Concepts in Graph Theory*
The notation $\mathcal{P}_k(V)$ stands for the set of all k-element subsets of the set V. Based on the previous example we have	*The notation $\mathcal{P}_k(V)$ stands for the set of all k-element subsets of the set V. Based on the previous example we have*
Definition 1 (Simple graph) A simple graph G is a pair $G = (V, E)$ where	*Definition 1 (Simple graph) A simple graph G is a pair $G = (V, E)$ where*
- V is a finite set, called the vertices of G, and *- E is a subset of $\mathcal{P}_2(V)$ (i.e., a set E of two-element subsets of V), called the edges of G.*	*- V is a finite set, called the vertices of G, and* *- E is a subset of $\mathcal{P}_2(V)$ (i.e., a set E of two-element subsets of V), called the edges of G.*
...	*...*

Table 12: Duplication matches found in Textbooks by MinHash LSH deduplication (in *italics*).

This was originally posted on mathoverflow, but it seems it's more appropriate to post here. *Let B be a paracompact space with the property that any (topological) vector bundle $E \to B$ is trivial. What are some non-trivial examples of such spaces, and are there any interesting properties that characterize them?* *For simple known examples we of course have contractible spaces, as well as the 3-sphere S^3. This one follows from the fact that its rank n vector bundles are classified by $\pi_3(BO(n)) = \pi_2(O(n)) = 0$. I'm primarily interested in the case where B is a closed manifold. Do we know any other such examples?* *There is this nice answer to a MSE question which talks about using the Whitehead tower of the appropriate classifying space to determine whether a bundle is trivial or not. This seems like a nice tool (of which I am not familiar with) to approaching this problem. As a secondary question, could I ask for some insight/references to this approach?* *EDIT Now that we know from the answer all the examples for closed 3-manifolds (integral homology spheres), I guess I can now update the question to the case of higher odd dimensions. Does there exist a higher dimensional example?*	*Let B be a paracompact space with the property that any (topological) vector bundle $E \to B$ is trivial. What are some non-trivial examples of such spaces, and are there any interesting properties that characterize them?* *For simple known examples we of course have contractible spaces, as well as the 3-sphere S^3. This one follows from the fact that its rank n vector bundles are classified by $\pi_3(BO(n)) = \pi_2(O(n)) = 0$. I'm primarily interested in the case where B is a closed manifold. Do we know any other such examples?* *There is this nice answer to a MSE question which talks about using the Whitehead tower of the appropriate classifying space to determine whether a bundle is trivial or not. This seems like a nice tool (of which I am not familiar with) to approaching this problem. As a secondary question, could I ask for some insight/references to this approach?* *EDIT Now that we know from the answers all the examples for closed 3-manifolds, I guess I can now update the question to the case of higher odd dimensions. Does there exist a higher dimensional example?*
This is a copy of my question on MSE (https://math.stackexchange.com/questions/3372432) because this forum seems better suited for historical questions: *In 1985, Gosper used the not-yet-proven formula by Ramanujan* $\frac{1}{\pi} = \frac{2\sqrt{2}}{99^2} \cdot \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \cdot \frac{26390n + 1103}{396^{4n}}$ *to compute $17 \cdot 10^6$ digits of π, at that time a new world record.* *Here (https://www.cs.princeton.edu/courses/archive/fall98/cs126/refs/pi-ref.txt) it reads:* *There were a few interesting things about Gosper's computation. First, when he decided to use that particular formula, there was no proof that it actually converged to pi! Ramanujan never gave the math behind his work, and the Borweins had not yet been able to prove it, because there was some very heavy math that needed to be worked through. It appears that Ramanujan simply observed the equations were converging to the 1103 in the formula, and then assumed it must actually be 1103. (Ramanujan was not known for rigor in his math, or for providing any proofs or intermediate math in his formulas.) The math of the Borwein's proof was such that after he had computed 10 million digits, and verified them against a known calculation, his computation became part of the proof. Basically it was like, if you have two integers differing by less than one, then they have to be the same integer.* *Now my historical question: Who was the first to prove this formula? Was it Gosper because he added the last piece of the proof, or was it the Borweins, afterwards? And was Gosper aware of this proof when he did his computation?*	*In 1985, Gosper used the not-yet-proven formula by Ramanujan* $\frac{1}{\pi} = \frac{2\sqrt{2}}{99^2} \cdot \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \cdot \frac{26390n + 1103}{99^{4n}}$ *to compute $17 \cdot 10^6$ digits of π, at that time a new world record.* *Here (https://www.cs.princeton.edu/courses/archive/fall98/cs126/refs/pi-ref.txt) it reads:* *There were a few interesting things about Gosper's computation. First, when he decided to use that particular formula, there was no proof that it actually converged to pi! Ramanujan never gave the math behind his work, and the Borweins had not yet been able to prove it, because there was some very heavy math that needed to be worked through. It appears that Ramanujan simply observed the equations were converging to the 1103 in the formula, and then assumed it must actually be 1103. (Ramanujan was not known for rigor in his math, or for providing any proofs or intermediate math in his formulas.) The math of the Borwein's proof was such that after he had computed 10 million digits, and verified them against a known calculation, his computation became part of the proof. Basically it was like, if you have two integers differing by less than one, then they have to be the same integer.* *Now my historical question: Who was the first to prove this formula? Was it Gosper because he added the last piece of the proof, or was it the Borweins, afterwards? And was Gosper aware of this proof when he did his computation?*

Table 13: Near-duplication matches found in StackExchange by MinHash LSH deduplication (in *italics*).

Coin A is flipped three times and coin B is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?

Video Solution

Answer:

## Problem 3.2.2 (AMC 10)

Two tour guides are leading six tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the stipulation that each guide must take at least one tourist. How many different groupings of guides and tourists are possible?

......

One morning each member of Angela’s family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

Answer:

## Problem 20.2.15 (AMC 12)

The state income tax where Kristin lives is levied at the rate of $p\%$ of the first $\$28000$ of annual income plus $(p+2)\%$ of any amount above $\$28000$. Kristin noticed that the state income tax she paid amounted to $(p+0.25)\%$ of her annual income. What was her annual income?

Answer:

......

Find the least positive integer k for which the equation $\left\lfloor \frac{2002}{n} \right\rfloor = k$ has no integer solutions for n. (The notation $\lfloor x \rfloor$ means the greatest integer less than or equal to x.)

Answer:

## Problem 40.1.9 (AIME)

Find the number of positive integers n less than 1000 for which there exists a positive real number x such that $n = x\lfloor x \rfloor$.’, ”, ’Note: $\lfloor x \rfloor$ is the greatest integer less than or equal to x.’

......

What is the sum of the roots of $z^{12} = 64$ that have a positive real part?

Answer:

## Problem 45.8.13 (AMC 12)

The complex numbers z and w satisfy $z^{13} = w, w^{11} = z$, and the imaginary part of z is $\sin \frac{m\pi}{n}$, for relatively prime positive integers m and n with $m < n$. Find n.’

Answer:

......

Table 14: Exact match examples from the test set of MATH benchmark found in Textbooks by line-level exact match deduplication (in *italics*).

Let x and y be real numbers satisfying $x^4y^5 + y^4x^5 = 810$ and $x^3y^6 + y^3x^6 = 945$. Evaluate $2x^3 + (xy)^3 + 2y^3$.
Let $x_1 < x_2 < x_3$ be the three real roots of the equation $\sqrt{2014}x^3 - 4029x^2 + 2 = 0$. Find $x_2(x_1 + x_3)$.
Let m be the largest real solution to the equation

$$\frac{3}{x-3} + \frac{5}{x-5} + \frac{17}{x-17} + \frac{19}{x-19} = x^2 - 11x - 4$$

There are positive integers a, b, and c such that $m = a + \sqrt{b + \sqrt{c}}$. Find $a + b + c$.
Let $f(x) = x^4 + ax^3 + bx^2 + cx + d$. If $f(-1) = -1$, $f(2) = -4$, $f(-3) = -9$, and $f(4) = -16$. Find $f(1)$.
Solve in positive integers $x^2 - 4xy + 5y^2 = 169$.
Solve in integers the question $x + y = x^2 - xy + y^2$.
Solve in integers $\frac{x+y}{x^2-xy+y^2} = \frac{3}{7}$
Prove the product of 4 consecutive positive integers is a perfect square minus 1.
For any arithmetic sequence whose terms are all positive integers, show that if one term is a perfect square, this sequence must have infinite number of terms which are perfect squares.
Prove there exist infinite number of positive integer a such that for any positive integer n, $n^4 + a$ is not a prime number.

......

The real root of the equation $8x^3 - 3x^2 - 3x - 1 = 0$ can be written in the form $\frac{\sqrt[3]{a}+\sqrt[3]{b}+1}{c}$, where a, b, and c are posit ive integers. Find $a + b + c$.
Find the number of positive integers m for which there exist nonnegative integers $x_0, x_1, \ldots, x_{2011}$ such that

$$m^{x_0} = \sum_{k=1}^{2011} m^{x_k}.$$

Suppose x is in the interval $[0, \frac{\pi}{2}]$ and $\log_{24 \sin x}(24 \cos x) = \frac{3}{2}$. Find $24 \cot^2 x$.
Let $P(x)$ be a quadratic polynomial with real coefficients satisfying $x^2 - 2x + 2 \leq P(x) \leq 2x^2 - 4x + 3$ for all real numbers x, and suppose $P(11) = 181$. Find $P(16)$.
Let (a, b, c) be the real solution of the system of equations $x^3 - xyz = 2$, $y^3 - xyz = 6$, $z^3 - xyz = 20$. The greatest possible value of $a^3 + b^3 + c^3$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
Find the smallest positive integer n with the property that the polynomial $x^4 - nx + 63$ can be written as a product of two nonconstant polynomials with integer coefficients.
The zeros of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?
Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x - a)(x - b) + (x - b)(x - c) = 0$?
At the theater children get in for half price. The price for 5 adult tickets and 4 child tickets is 24.50. How much would 8 adult tickets and 6 child tickets cost?
The quadratic equation $x^2 + px + 2p = 0$ has solutions $x = a$ and $x = b$. If the quadratic equation $x^2 + cx + d = 0$ has solutions $x = a + 2$ and $x = b + 2$, what is the value of d?

......

Find the smallest positive integer n with the property that the polynomial $x^4 - nx + 63$ can be written as a product of two nonconstant polynomials with integer coefficients.
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The quadratic equation $x^2 + px + 2p = 0$ has solutions $x = a$ and $x = b$. If the quadratic equation $x^2 + cx + d = 0$ has solutions $x = a + 2$ and $x = b + 2$, what is the value of d?
PolynomialAndEquation Root Delta SpecialEquation Function NumberTheoryBasic IndeterminateEquation SqueezeMethod Pythagore anTripletFormula TrigIdentity Inequality LogicalAndReasoning
AMC10/12 AIME IMO
US International
With Solutions
© 2009 - 2023 Math All Star

......

Table 15: Exact match examples from the test set of MATH benchmark found in CommonCrawl by line-level exact match deduplication (in *italics*). In these examples, we only observe repeated questions from MATH, but do not identify duplicate answers.

The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

(A) $\frac{1+\sqrt{5}}{2}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** 4

## Problem 17

All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

(A) 312 **(B)** 343 **(C)** 625 **(D)** 729 **(E)** 1680

...

What is the value of $b + c$ if $x^2 + bx + c > 0$ only when $x \in (-\infty, -2) \cup (3, \infty)$?

May 11, 2020

...

An ambulance travels at 40 mph and can follow a 20-mile route making no stops to get to the hospital. A helicopter travels at one mile per minute, and the air route is 15 miles to get to the same hospital. However, the helicopter takes three minutes for takeoff and three minutes for landing. How many fewer minutes does it take for the helicopter to complete its trip (takeoff, flight and landing) than for the ambulance to complete its trip?

Apr 6, 2020

#1
+34
0

Keep in mind that Time=Distance/Speed

What is the greatest possible area of a triangular region with one vertex at the center of a circle of radius 1 and the other two vertices on the circle?

A bad first step is to put the center at the origin, one point at (1,0) , and one point at (sin x, cos x).

A start is the area of a triangle with included angle expression

$$\frac{a \times b \times \sin\theta}{2}$$

Assuming θ in radians. If theta is $\pi/2$ then we have a right triangle. Let a=b=1. Area expression is

$$A = (\sin\theta)/2$$

This is maximum for $\theta = \pi/2$.

Answer is maximum area for a right triangle.

...

Table 16: Exact match examples from the test set of MATH benchmark (upper) and MMLU-STEM (bottom) found in OpenWebMath by line-level exact match deduplication (in *italics*). In these examples, we only observe repeated questions, but do not identify duplicate answers.