

$$\nabla_v \tilde{S}_y \left( \begin{array}{l} h_x(-2r_4/(1+(\sigma((-2r_2r_4\cos((\theta_2)_j)+2r_1r_4\cos(\theta_1))^2+(-2r_2r_4\sin((\theta_2)_j)+2r_1r_4\sin(\theta_1))^2) \\ \vdots \end{array} \right.$$

$$\nabla_v \tilde{S}_y \left( \begin{array}{l} -h_x(2r_4/(1+(\sigma((-2r_2r_4\cos((\theta_2)_j)+2r_1r_4\cos(\theta_1))^2+(-2r_2r_4\sin((\theta_2)_j)+2r_1r_4\sin(\theta_1))^2) \\ \vdots \end{array} \right.$$

$$\frac{\partial \tilde{S}_x}{\partial (\theta_2)_j} = h_y(r_2\cos((\theta_2)_j)-2r_4/(1+(\sigma((-2r_2r_4\cos((\theta_2)_j)+2r_1r_4\cos(\theta_1))^2+(-2r_2r_4\sin((\theta_2)_j)+2r_1r_4\sin(\theta_1))^2)$$

$$\frac{\partial \tilde{S}_y}{\partial (\theta_2)_j} = h_y(-r_2\sin((\theta_2)_j)+2r_4/(1+(\sigma((-2r_2r_4\cos((\theta_2)_j)+2r_1r_4\cos(\theta_1))^2+(-2r_2r_4\sin((\theta_2)_j)+2r_1r_4\sin(\theta_1))^2)$$