EECS 592: Bayes Net Example, Mar. 21, 2018

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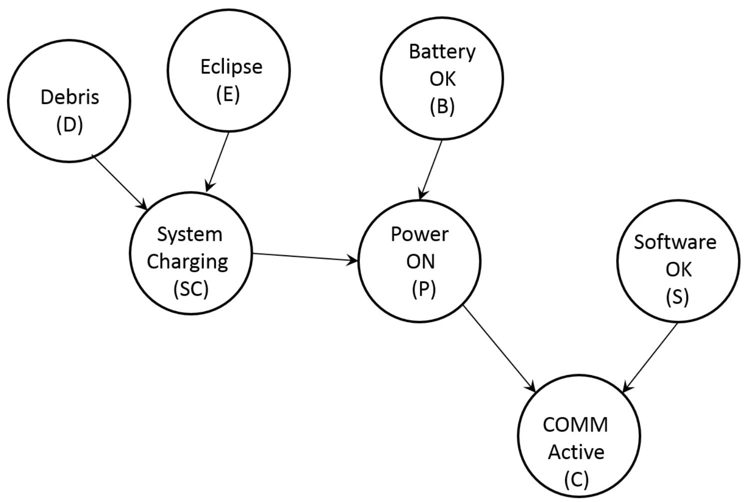

Figure 1: Example Bayesian Network.

Network Probabilities: $P(D) = 0.1; P(E) = 0.3; P(B) = 0.95; P(S) = 0.99$
$P(SC|D, E) = 0.0; P(SC|D, \neg E) = 0.6; P(SC|\neg D, E) = 0.0; P(SC|\neg D, \neg E) = 0.9$
$P(P|SC, B) = 0.9; P(P|SC, \neg B) = 0.1; P(P|\neg SC, B) = 0.7; P(P|\neg SC, \neg B) = 0.0$
$P(C|P, S) = 0.95; P(C|P, \neg S) = 0.5; P(C|\neg P, S) = 0.01; P(C|\neg P, \neg S) = 0.0$

1 Problem 1:

Which node(s) if any are conditionally independent of P given SC and C?

Ans: Evidence for node SC breaks the connection between P and D,E. Evidence for node C causes P to depend on node S. So, D and E are conditionally independent of P.

2 Problem 2:

Which node(s) if any are conditionally independent of B given SC?

Ans: Again, evidence for SC breaks the connection between the P side of the network and D,E. Node B is also independent of S because no evidence is given for C. Overall, nodes D, E, and S are conditionally independent of B.

3 Problem 3:

Compute $P(SC|E)$

Ans:

$$P(SC|E) = P(SC|D,E)P(D|E) + P(SC|\neg D,E)P(\neg D|E)$$

Because D and E are parents of P, and no other (downstream) evidence is given in $P(D|E)$, D is conditionally independent of E such that $P(D|E) = P(D)$. A similar answer is given for the negation term. This produces a final answer of:

$$P(SC|E) = P(SC|D,E)P(D) + P(SC|\neg D,E)P(\neg D) = 0.$$

4 Problem 4:

Compute $P(SC|P)$

Ans:

$$P(SC|P) = \frac{P(P|SC)P(SC)}{P(P)}$$

$$P(P|SC) = P(P|SC,B)P(B|SC) + P(P|SC,\neg B)P(\neg B|SC)$$

$$= P(P|SC,B)P(B) + P(P|SC,\neg B)P(\neg B) = 0.86$$

Above, we also note that B is conditionally independent of SC since B and SC are parent nodes of P, and P plus all other nodes downstream of P are not given as evidence in $P(B|SC)$.

Now compute marginal probabilities $P(SC)$ and $P(P)$. Note that D and E are conditionally independent so $P(D|E) = P(D)$, etc.

$$P(SC) = P(SC|D,E)P(D)P(E) + P(SC|D,\neg E)P(D)P(\neg E)$$

$$+P(SC|\neg D,E)P(\neg D)P(E) + P(SC|\neg D,\neg E)P(\neg D)P(\neg E) = 0.609$$

Similarly, $P(B|SC) = P(B)$ since B is conditionally independent of B.

$$P(P) = P(P|B,SC)P(B)P(SC) + P(P|B,\neg SC)P(B)P(\neg SC)$$

$$+P(P|\neg B,SC)P(\neg B)P(SC) + P(P|\neg B,\neg SC)P(\neg B)P(\neg SC) = 0.784$$

Finally:

$$P(SC|P) = \frac{P(P|SC)P(SC)}{P(P)} = \frac{0.86 * 0.609}{0.784} - 0.668$$

5 Problem 5 (from real-time lecture (!)):

Compute $P(P|SC,C)$.

Ans:

Steps to multivariable Bayes Rule application:

$$P(P,SC,C) = P(P|SC,C)P(SC,C) = P(P|C,SC)P(C,SC) = P(C|P,SC)P(P,SC)$$

$$P(P,SC) = P(P|SC)P(SC); P(C,SC) = P(C|SC)P(SC)$$

$$P(P|SC,C) = P(P|C,SC)$$

$$P(P|C,SC) = \frac{P(C|P,SC)P(P,SC)}{P(C,SC)}$$

$$P(P|C,SC) = \frac{P(C|P,SC)P(P|SC)P(SC)}{P(C|SC)P(SC)}$$

$$P(P|SC,C) = P(P|C,SC) = \frac{P(C|P,SC)P(P|SC)}{P(C|SC)}$$

Now compute the three probabilities from this Bayes Rule expression. From Problem 4, we computed

$$P(P|SC) = 0.86$$

To compute $P(C|P,SC)$, note that C is conditionally independent of SC given P:

$$P(C|P,SC) = P(C|P) = P(C|P,S)P(S) + P(C|P,\neg S)P(\neg S) = 0.9455$$

Next,

$$P(C|SC) = P(C|SC,P)P(P|SC) + P(C|SC,\neg P)P(\neg P|SC)$$

In Problem 4, we computed $P(P) = 0.784$ so $P(\neg P) = 1 - 0.784 = 0.216$. Also, $P(C|SC,P) = P(C|P,SC) = 0.9455$, and :

$$P(C|SC,\neg P) = P(C|\neg P) = P(C|\neg P,S)P(S) + P(C|\neg P,\neg S)P(\neg S) = 0.0099$$

$$P(C|SC) = 0.9455 * 0.86 + 0.0099 * (1 - 0.86) = 0.8145$$

Finally,

$$P(P|SC,C) = \frac{(0.9455)(0.86)}{(0.8145)} = 0.9983$$