

# Language Models as Hierarchy Encoders

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## Motivation

### Existing pre-trained LMs lack explicit hierarchy interpretation

- Pre-trained LMs can predict relations like “A is B” and “B is C” but struggle to infer the **transitive relationship** “A is C” [1]
- These models typically encode hierarchical entities based on **similarities** rather than structural relationships [2]

### Limitations of existing hyperbolic embeddings

- Classic hyperbolic embeddings, such as Poincare Embeddings [3] and Hyperbolic Entailment Cone [4], are **static** and only capture hierarchy within a **fixed entity set**
- Hyperbolic word embeddings [5] face limitations due to **word-level tokenisation** and **unified word representations**

## Preliminaries

### Hyperbolic Geometry

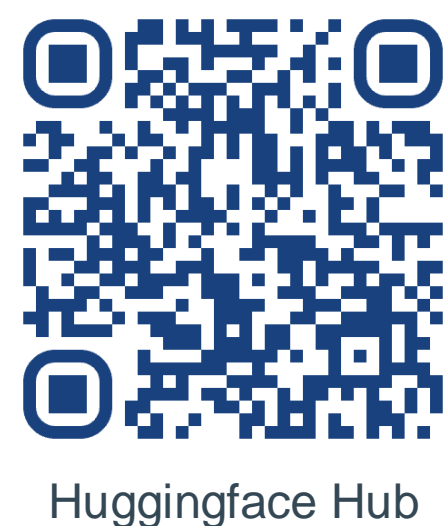
- A form of **non-Euclidean geometry** characterised by its constant negative Gaussian curvature
- The distance between points grows **exponentially** as they approach the manifold’s boundary
- Provides a **theoretical guarantee** for embedding tree-like structures [4]
- Poincaré ball**: A  $d$ -dimensional open ball  $\mathbb{B}_c^d = \{\mathbf{x} \in \mathbb{R}^d: \|\mathbf{x}\|^2 < \frac{1}{c}\}$
- Distance function**:  $d_c(\mathbf{u}, \mathbf{v}) = \frac{2}{\sqrt{c}} \tanh^{-1}(\sqrt{c} \|\mathbf{u} \oplus_c \mathbf{v}\|)$  where  $\oplus_c$  denotes the Möbius addition.

### Hierarchy

- A directed acyclic graph  $\mathcal{G}(\mathcal{V}, \mathcal{E})$  where  $\mathcal{V}$  represents **entities** as vertices and  $\mathcal{E}$  represents **direct subsumption relationships** as edges
- Indirect subsumptions**  $\mathcal{T}$  are derived from *transitive reasoning*
- Negative subsumptions** are  $(e_1 \in \mathcal{E}, e_2 \in \mathcal{E}) \notin \mathcal{E} \cup \mathcal{T}$  (closed-world assumption)

### References

- [1] Lin et al. “Does bert know that the is-a relation is transitive?” In: ACL 2022.  
 [2] Liu et al. “Self-alignment pretraining for biomedical entity representations” In: NAACL 2021.  
 [3] Nickel et al. “Poincaré embeddings for learning hierarchical representations” In: NeurIPS 2017.  
 [4] Ganea et al. “Hyperbolic entailment cones for learning hierarchical embeddings” In: ICML 2018.  
 [5] Tifrea et al. “Poincare glove: Hyperbolic word embeddings.” In: ICLR 2018.



## Hierarchy Transformer Encoder (HiT)

### Construction

- The output embedding space of transformer encoder-based LMs is often a  **$d$ -dimensional hyper-cube** due to the tanh activation function in the last layer. We can then construct a **Poincaré ball** of radius  $\sqrt{d}$  (a  **$d$ -dimensional hyper-sphere**) so that its boundary **circumscribes** the output embedding space of LMs
- We utilise the **sentence transformer architecture** except that we **exclude the normalisation layer** after mean pooling over token embeddings as it prevents hierarchical organisation

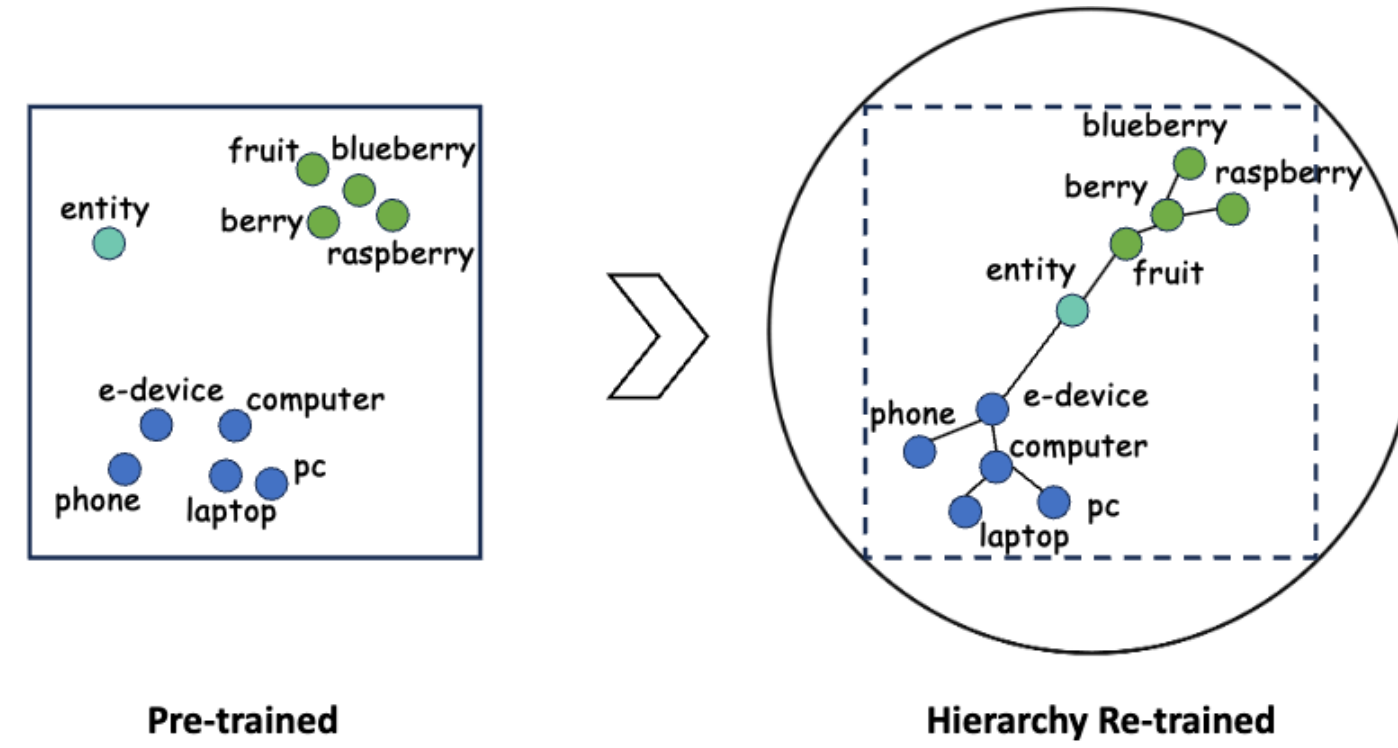


Fig 1. Illustration of how hierarchies are explicitly encoded in **HiT**.

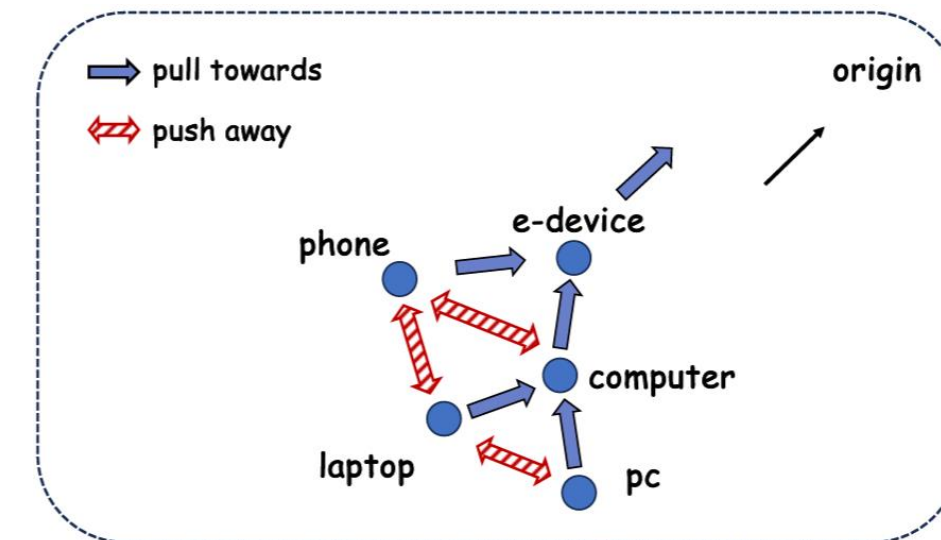


Fig 2. Illustration of the impact of hyperbolic clustering and centripetal losses.

### Hyperbolic Losses

- Hyperbolic Clustering Loss**: to cluster related entities while distancing unrelated ones

$$\mathcal{L}_{cluster} = \sum_{(e, e^+, e^-) \in \mathcal{D}} \max(d_c(\mathbf{e}, \mathbf{e}^+) - d_c(\mathbf{e}, \mathbf{e}^-) + \alpha, 0)$$

- Hyperbolic Centripetal Loss**: to position the parent entities closer to the manifold’s origin than child counterparts

$$\mathcal{L}_{centri} = \sum_{(e, e^+, e^-) \in \mathcal{D}} \max(\|\mathbf{e}^+\| - \|\mathbf{e}\| + \beta, 0)$$

- The overall loss is the linear combination of the above two losses.
- Subsumption Prediction Function**: to probe the resulting **HiT** model to predict entity subsumptions

$$s(e_1 \sqsubseteq e_2) = -(d_c(\mathbf{e}_1, \mathbf{e}_2) + \lambda(\|\mathbf{e}_2\|_c - \|\mathbf{e}_1\|_c))$$

## Evaluation

### Task Definition

- Multi-hop Inference**: Trained on asserted (one-hop) subsumptions and tested on transitively inferred (multi-hop) subsumptions
- Mixed-hop Prediction**: Trained on incomplete asserted subsumptions and tested on arbitrary, probably unseen subsumptions
- Mixed-hop Prediction (Transfer)**: Trained on asserted subsumptions of one hierarchy and tested on arbitrary subsumptions of another hierarchy
- Evaluation Metrics**: Precision, Recall, and F-score

### Dataset

| Source     | #Entity | #DirectSub | #IndirectSub | #Dataset (Train/Val/Test)                      |
|------------|---------|------------|--------------|--|
| WordNet    | 74,401  | 75,850     | 587,658      | multi: 834K/323K/323K<br>mixed: 751K/365K/365K |
| Schema.org | 903     | 950        | 1,978        | mixed: -/15K/15K                               |
| FoodOn     | 30,963  | 36,486     | 438,266      | mixed: 361K/261K/261K                          |
| DOID       | 11,157  | 11,180     | 45,383       | mixed: 122K/31K/31K                            |
| SNOMED     | 364,352 | 420,193    | 2,775,696    | mixed: 4,160K/1,758K/1,758K                    |

### Results

| Model  | Random Negatives |        |         | Hard Negatives |        |         |
|--|------------------|--------|---------|----------------|--------|---------|
|  | Precision        | Recall | F-score | Precision      | Recall | F-score |
| NaivePrior                                     | 0.091            | 0.091  | 0.091   | 0.091          | 0.091  | 0.091   |
| Multi-hop Inference (WordNet)                  |                  |        |         |                |        |         |
| PoincaréEmbed                                  | 0.862            | 0.866  | 0.864   | 0.797          | 0.867  | 0.830   |
| HyperbolicCone                                 | 0.817            | 0.996  | 0.898   | 0.243          | 0.902  | 0.383   |
| all-MiniLM-L12-v2                              | 0.127            | 0.585  | 0.209   | 0.108          | 0.740  | 0.188   |
| + fine-tune                                    | 0.811            | 0.515  | 0.630   | 0.819          | 0.530  | 0.643   |
| + HiT  | 0.880            | 0.927  | 0.903   | 0.910          | 0.906  | 0.908   |
| Mixed-hop Prediction (WordNet)                 |                  |        |         |                |        |         |
| all-MiniLM-L12-v2                              | 0.127            | 0.583  | 0.209   | 0.111          | 0.625  | 0.188   |
| + fine-tune                                    | 0.794            | 0.517  | 0.627   | 0.859          | 0.515  | 0.644   |
| + HiT  | 0.875            | 0.895  | 0.885   | 0.886          | 0.857  | 0.871   |
| Transfer Mixed-hop Prediction (WordNet → DOID) |                  |        |         |                |        |         |
| PoincaréGloVe                                  | 0.265            | 0.314  | 0.287   | 0.283          | 0.318  | 0.299   |
| all-MiniLM-L12-v2                              | 0.342            | 0.451  | 0.389   | 0.159          | 0.455  | 0.235   |
| + fine-tune                                    | 0.585            | 0.621  | 0.603   | 0.868          | 0.179  | 0.297   |
| + HiT  | 0.696            | 0.711  | 0.704   | 0.810          | 0.435  | 0.566   |

### Analysis

- The hyperbolic norms of entity embeddings in **HiT** capture the natural expansion of hierarchies
- HiT** demonstrates a stronger linear relationship between entity hyperbolic norms and depths

### Future Work

- Mitigate catastrophic forgetting
- Develop **hierarchy-based** semantic search

