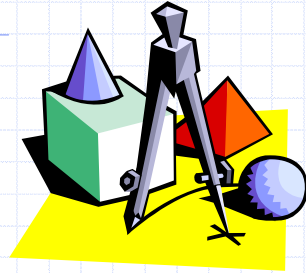


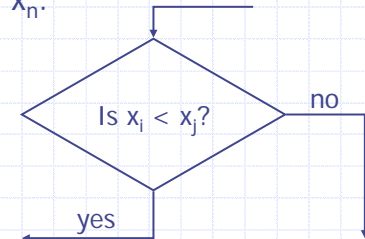
Sorting Lower Bound



Comparison-Based Sorting

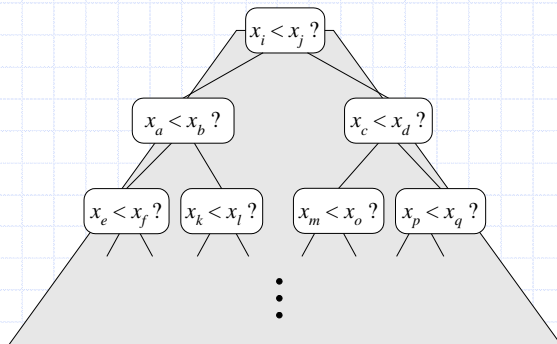


- Many sorting algorithms are comparison based.
 - They sort by making comparisons between pairs of objects
 - Examples: bubble-sort, selection-sort, insertion-sort, heap-sort, merge-sort, quick-sort, ...
- Let us therefore derive a lower bound on the running time of any algorithm that uses comparisons to sort n elements, x_1, x_2, \dots, x_n .



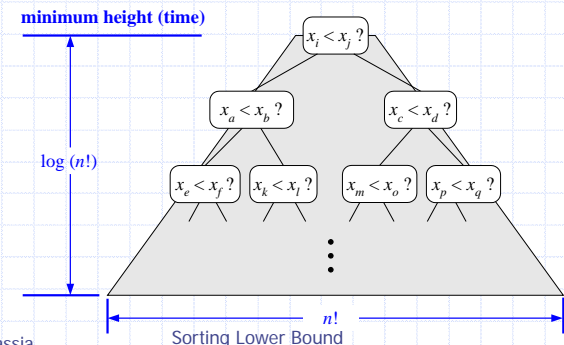
Counting Comparisons

- Let us just count comparisons then.
- Each possible run of the algorithm corresponds to a root-to-leaf path in a **decision tree**

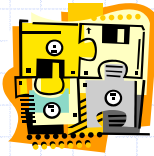


Decision Tree Height

- The height of the decision tree is a lower bound on the running time
- Every input permutation must lead to a separate leaf output
- If not, some input ...4...5... would have same output ordering as ...5...4..., which would be wrong
- Since there are $n! = 1 \cdot 2 \cdot \dots \cdot n$ leaves, the height is at least $\log(n!)$



The Lower Bound



- ◆ Any comparison-based sorting algorithm takes at least $\log(n!)$ time
- ◆ Therefore, any such algorithm takes time at least

$$\log(n!) \geq \log\left(\frac{n}{2}\right)^{\frac{n}{2}} = (n/2) \log(n/2).$$

- ◆ That is, any comparison-based sorting algorithm must run in $\Omega(n \log n)$ time.