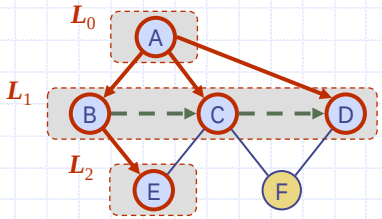
Breadth-First Search



Breadth-First Search

- Breadth-first search (BFS) is a general technique for traversing a graph
- A BFS traversal of a graph G
  - Visits all the vertices and edges of G
  - Determines whether G is connected
  - Computes the connected components of G
  - Computes a spanning forest of G
- BFS on a graph with n vertices and m edges takes $O(n + m)$ time
- BFS can be further extended to solve other graph problems
  - Find and report a path with the minimum number of edges between two given vertices
  - Find a simple cycle, if there is one

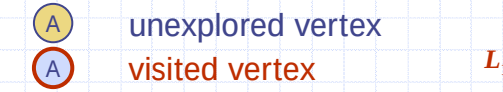
BFS Algorithm

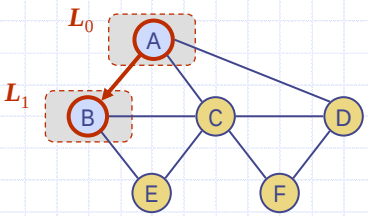
- The algorithm uses a mechanism for setting and getting "labels" of vertices and edges

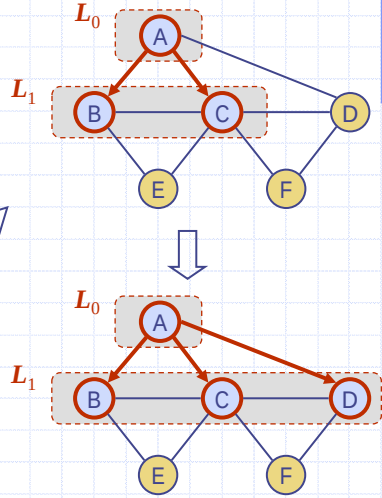
```
Algorithm BFS(G)
    Input graph G
    Output labeling of the edges
        and partition of the
        vertices of G
  for all u ∈ G.vertices()
    u.setLabel(UNEXPLORED)
  for all e ∈ G.edges()
    e.setLabel(UNEXPLORED)
  for all v ∈ G.vertices()
    if v.getLabel() = UNEXPLORED
      BFS(G, v)
```

```
Algorithm BFS(G, s)
  L0 ← new empty sequence
  L0.insertBack(s)
  s.setLabel(VISITED)
  i ← 0
  while ¬Li.empty()
    Li+1 ← new empty sequence
    for all v ∈ Li.elements()
      for all e ∈ v.incidentEdges()
        if e.getLabel() = UNEXPLORED
          w ← e.opposite(v)
          if w.getLabel() = UNEXPLORED
            e.setLabel(DISCOVERY)
            w.setLabel(VISITED)
            Li+1.insertBack(w)
          else
            e.setLabel(CROSS)
    i ← i +1
```

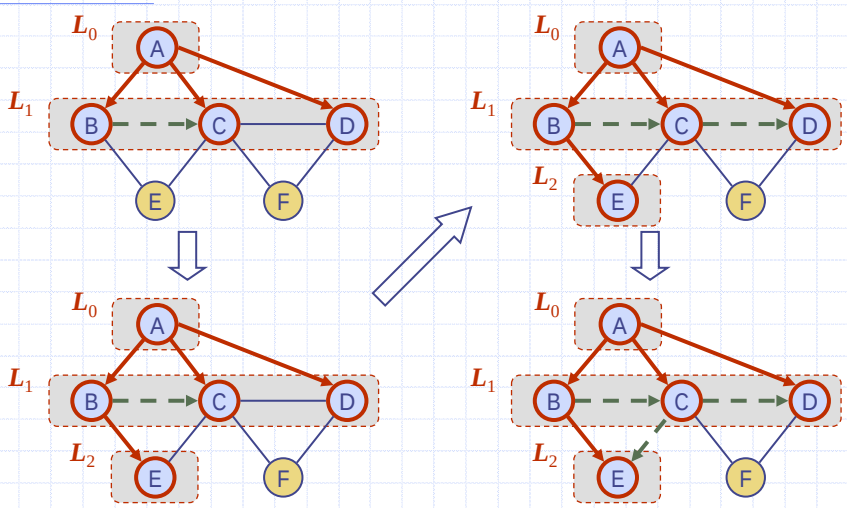

Example



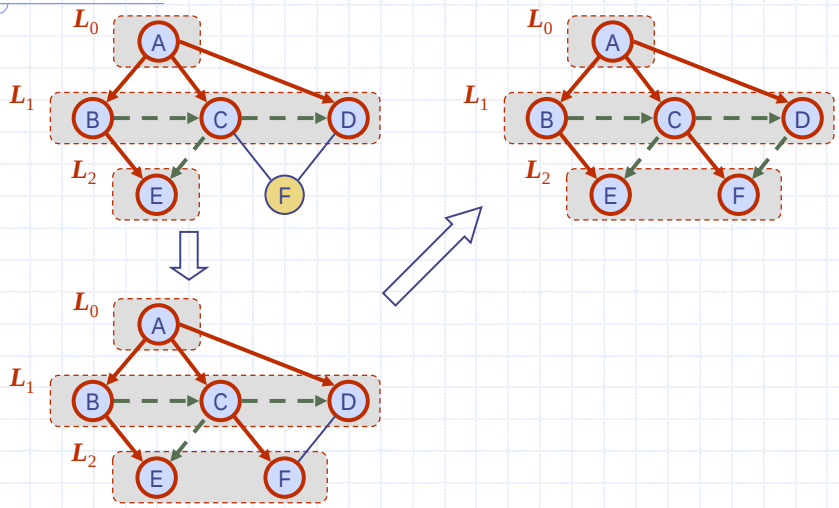




Example (cont.)



Example (cont.)



Properties

Notation

G_s: connected component of s

Property 1

$BFS(G, s)$ visits all the vertices and edges of G_s

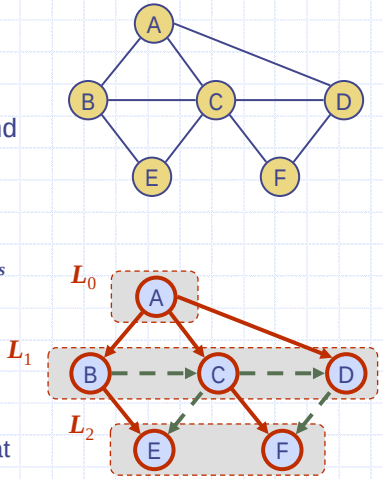
Property 2

The discovery edges labeled by $BFS(G, s)$ form a spanning tree T_s of G_s

Property 3

For each vertex v in L_i

- The path of T_s from s to v has i edges
- Every path from s to v in G_s has at least i edges



Analysis

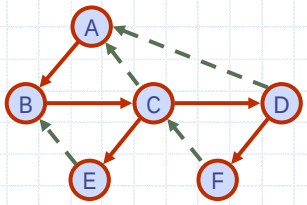
- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- Each vertex is inserted once into a sequence L_i
- Method incidentEdges is called once for each vertex
- BFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\sum_v \deg(v) = 2m$

Applications

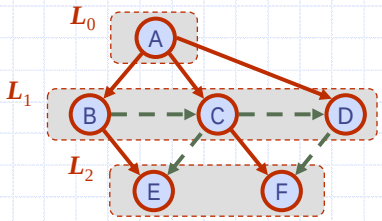
- Using the template method pattern, we can specialize the BFS traversal of a graph G to solve the following problems in $O(n + m)$ time
  - Compute the connected components of G
  - Compute a spanning forest of G
  - Find a simple cycle in G, or report that G is a forest
  - Given two vertices of G, find a path in G between them with the minimum number of edges, or report that no such path exists

DFS vs. BFS

Applications	DFS	BFS
Spanning forest, connected components, paths, cycles	√	√
Shortest paths		√
Biconnected components	√	



DFS

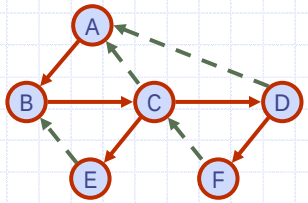


BFS

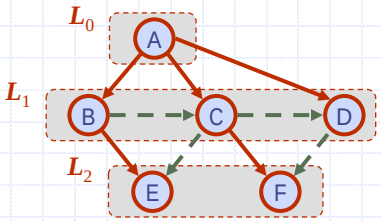
DFS vs. BFS (cont.)

Back edge (v,w)
- w is an ancestor of v in the tree of discovery edges

Cross edge (v,w)
- w is in the same level as v or in the next level



DFS



BFS