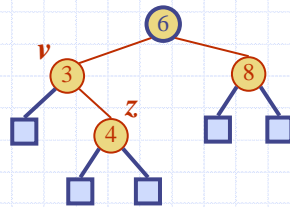
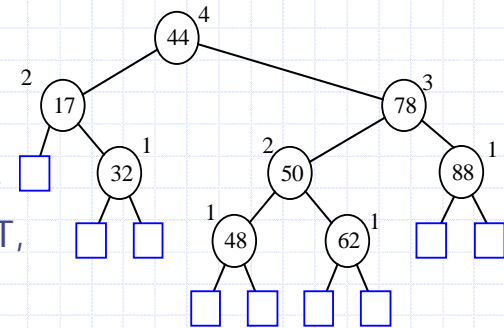


AVL Trees



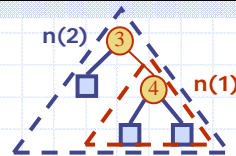
AVL Tree Definition

- ◆ AVL trees are balanced
- ◆ An AVL Tree is a **binary search tree** such that for every internal node v of T , the **heights of the children of v** can differ by at most 1



An example of an AVL tree where the heights are shown next to the nodes:

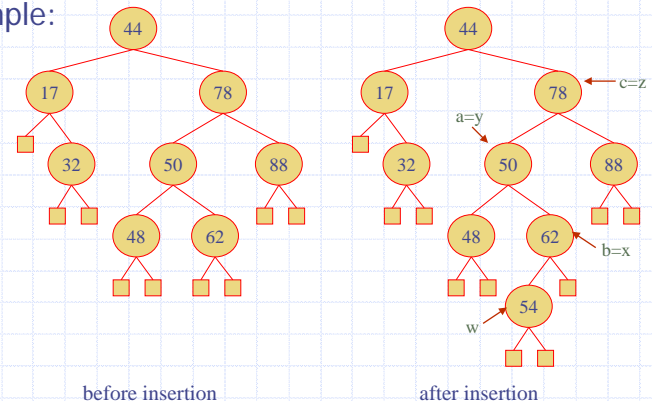
Height of an AVL Tree



- ◆ **Fact:** The **height** of an AVL tree storing n keys is $O(\log n)$.
- ◆ **Proof:** Let us bound $n(h)$: the minimum number of internal nodes of an AVL tree of height h .
- ◆ We easily see that $n(1) = 1$ and $n(2) = 2$
- ◆ For $n > 2$, an AVL tree of height h contains the root node, one AVL subtree of height $n-1$ and another of height $n-2$.
- ◆ That is, $n(h) = 1 + n(h-1) + n(h-2)$
- ◆ Knowing $n(h-1) > n(h-2)$, we get $n(h) > 2n(h-2)$. So
 $n(h) > 2n(h-2)$, $n(h) > 4n(h-4)$, $n(h) > 8n(h-6)$, ... (by induction),
 $n(h) > 2^{\lfloor h/2 \rfloor} n(h-2\lfloor h/2 \rfloor)$
- ◆ Solving the base case we get: $n(h) > 2^{h/2-1}$
- ◆ Taking logarithms: $h < 2 \log n(h) + 2$
- ◆ Thus the height of an AVL tree is $O(\log n)$

Insertion

- ◆ Insertion is as in a binary search tree
- ◆ Always done by expanding an external node.
- ◆ Example:



before insertion

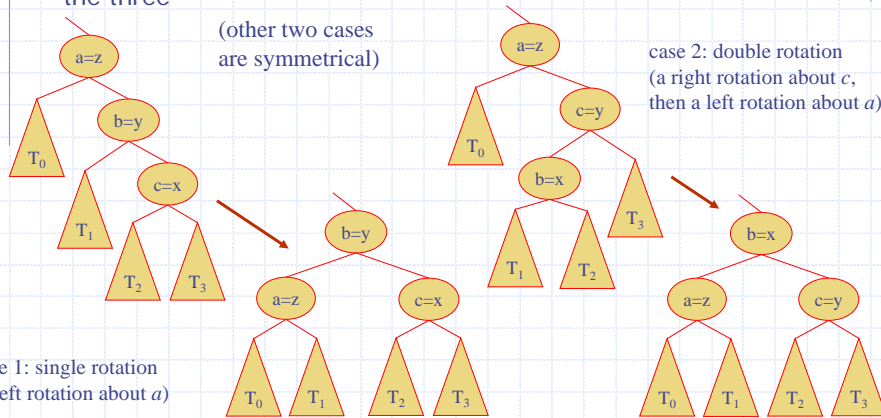
after insertion

Trinode Restructuring

- ◆ let (a, b, c) be an inorder listing of x, y, z
- ◆ perform the rotations needed to make b the topmost node of the three

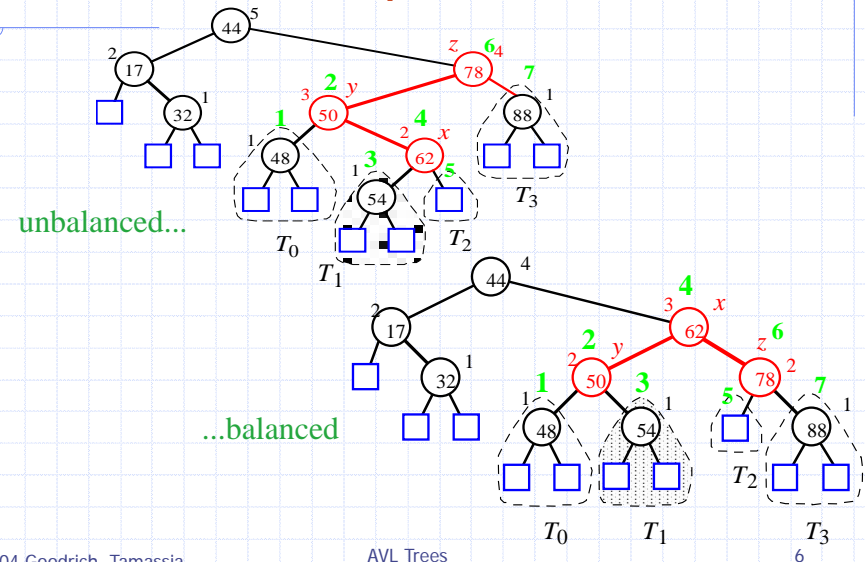
(other two cases are symmetrical)

case 2: double rotation
(a right rotation about c , then a left rotation about a)



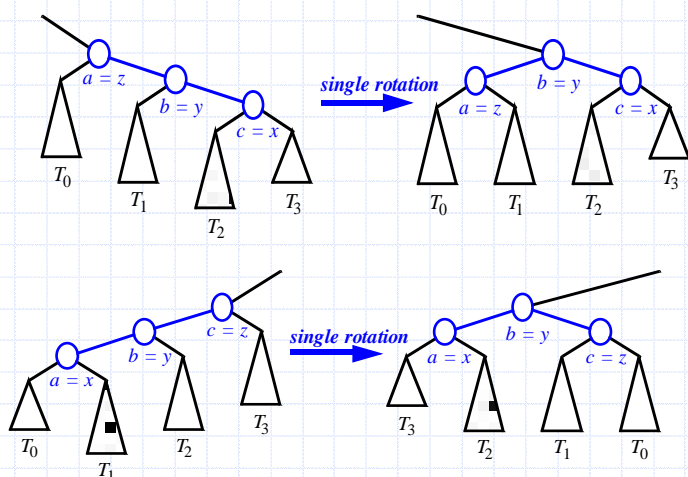
case 1: single rotation
(a left rotation about a)

Insertion Example, continued



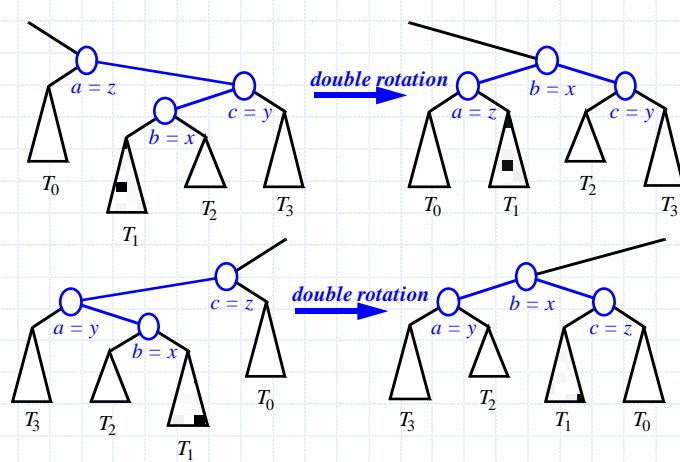
Restructuring (as Single Rotations)

- ◆ Single Rotations:



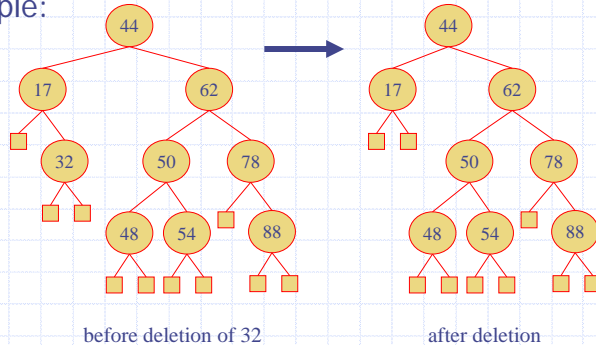
Restructuring (as Double Rotations)

- ◆ double rotations:



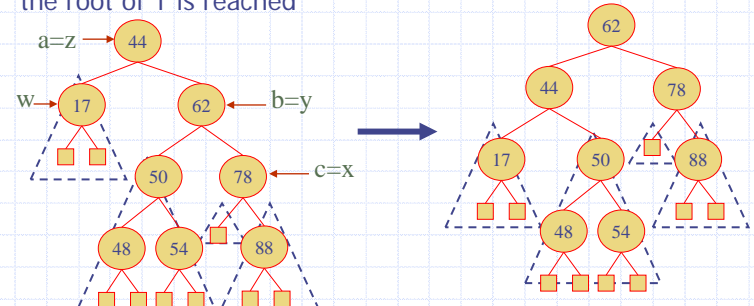
Removal

- ◆ Removal begins as in a binary search tree, which means the node removed will become an empty external node. Its parent, w , may cause an imbalance.
- ◆ Example:

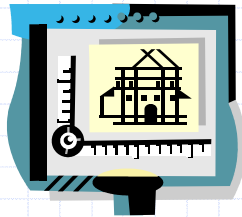


Rebalancing after a Removal

- ◆ Let z be the **first unbalanced** node encountered while travelling up the tree from w . Also, let y be the child of z with the larger height, and let x be the child of y with the larger height
- ◆ We perform **restructure**(x) to restore balance at z
- ◆ As this restructuring may upset the balance of another node higher in the tree, we must continue checking for balance until the root of T is reached



AVL Tree Performance



- ◆ a single restructure takes $O(1)$ time
 - using a linked-structure binary tree
- ◆ **find** takes $O(\log n)$ time
 - height of tree is $O(\log n)$, no restructures needed
- ◆ **put** takes $O(\log n)$ time
 - initial find is $O(\log n)$
 - Restructuring up the tree, maintaining heights is $O(\log n)$
- ◆ **erase** takes $O(\log n)$ time
 - initial find is $O(\log n)$
 - Restructuring up the tree, maintaining heights is $O(\log n)$